Formulario Electrodinámica

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I. RELATIVIDAT ESPECIAL

Transformación de Lorentz

$$x^{\mu'} = \Lambda^{\mu'}_{\nu} x^{\nu} \qquad \gamma(v) = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$ct' = \gamma\left(ct - \frac{v}{c}x\right) \quad x' = \gamma\left(x - vt\right)$$

$$v_{12} = \frac{v_1 + v_2}{1 + \frac{v_1 v_2}{c^2}}$$

$$t' = \gamma\left(t - \frac{\vec{v}\cdot\vec{r}}{c^2}\right) \quad \vec{r} = \vec{r} + \vec{v}\left[\frac{\vec{v}\cdot\vec{r}}{v^2}(\gamma - 1) - \gamma t\right]$$

$$\gamma_{u'} = \gamma_u \gamma_v \left(1 - \frac{\vec{u}\cdot\vec{v}}{c^2}\right)$$

Parametritzación Hiperbólica

$$\tanh\phi = \beta \quad \gamma = \cosh\phi \quad \gamma\beta = \sinh\phi$$

$$ct' = ct\cosh\phi - x\sinh\phi \quad x' = x\cosh\phi - ct\sinh\phi$$

Intérvalo

$$(\Delta s)^2 = (\Delta x^0)^2 - (\Delta\vec{x})^2 = (\Delta s')^2$$

Cinemática relativista

$$L = \gamma^{-1} L_0 \qquad T = \gamma T_0$$

$$\tan\alpha = \gamma\tan\alpha_0 \quad L = L_0\sqrt{1 - \beta^2\cos^2\alpha}$$

Transformación de velocidades:

$$u'^1 = \frac{u^1 - v}{1 - \frac{vu^1}{c^2}} \qquad u'^{\perp} = \frac{u^{\perp}}{\gamma\left(1 - \frac{vu^1}{c^2}\right)}$$

$$c^2 - \vec{u}^2 = c^2\gamma_u^{-2}$$

Aceleración propia:

$$\alpha = \gamma_u^3 \frac{\mathrm{d}u}{\mathrm{d}t}$$

II. ÓPTICA RELATIVISTA

$$\lambda = cT = \frac{c}{\nu}$$

Efecto Doppler

$$\lambda_R = \lambda_E \frac{1 - \vec{n}\cdot\vec{\beta}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Radial:

$$\lambda_R = \lambda_E \sqrt{\frac{c \mp v}{c \pm v}}$$

Transversal: $\lambda_R = \gamma\lambda_E$

Fórmula de fresnel

$$u = u' + kv \qquad k \equiv \left(1 - \frac{1}{n^2}\right) < 1$$

Aberración

$$\sin\alpha' = \frac{\sin\alpha}{\gamma(1 + \beta\cos\alpha)} \quad \cos\alpha' = \frac{\cos\alpha + \beta}{1 + \beta\cos\alpha}$$

III. DINÁMICA RELATIVISTA

$$\eta_{\mu\nu} = \eta^{\mu\nu} = \eta_{\mu'\nu'} = \mathrm{diag}(1, -1, -1, -1)$$

$$v_\mu = \eta_{\mu\nu} v^\nu \quad q^\mu = \eta^{\mu\nu} q_\nu \quad \eta_{\mu\nu} u^\mu v^\nu = u_\mu v^\mu = u^\mu v_\mu$$

$$\mathrm{d}t = \gamma\mathrm{d}\tau$$

Cuadrivelocidat

$$u^\mu = \frac{\mathrm{d}x^\mu}{\mathrm{d}\tau} = \gamma\frac{\mathrm{d}x^\mu}{\mathrm{d}t} = \gamma(c, \vec{u}) \qquad u^\mu u_\mu = c^2$$

$$u^4 = \sqrt{c^2 + \vec{u}^2} \quad \vec{\beta} = \frac{\vec{u}}{u^0}$$

Cuadriaceleración

$$a^\mu = \frac{\mathrm{d}u^\mu}{\mathrm{d}\tau} = \left(\frac{\gamma^4}{c}\vec{u}\cdot\vec{a}, \gamma^2\vec{a} + \frac{\gamma^4}{c^2}(\vec{a}\cdot\vec{u})\vec{u}\right)$$

$$\dot{\gamma} = \frac{\gamma^3}{c^2}\vec{u}\cdot\vec{a} \quad u^\mu a_\mu = 0$$

$$a^\mu a_\mu = -\left(\gamma^6\frac{u^2\dot{u}^2}{c^2} + \gamma^4 a^2\right)$$

SR comóbil: $a^{\mu'} = (0, \vec{\alpha})$, $u^{\mu'} = (c, \vec{0})$

$$a^\mu a_\mu = -\alpha^2 \qquad \alpha^2 = \gamma^6\left[a^2 - \frac{(\vec{u}\times\vec{a})^2}{c^2}\right]$$

Cuadrimomento

$$p^\mu = mu^\mu \quad E = m\gamma c^2 \quad p^\mu = \left(\frac{E}{c}, \vec{p}\right) \quad \vec{p} = m\gamma\vec{u}$$

$$p^\mu p_\mu = m^2c^2 \quad k^\mu = \frac{h\nu}{c}(1, \hat{k}) \quad k^\mu k_\mu = 0$$

$$T = E - mc^2 \quad E^2 = \vec{p}^2c^2 + m^2c^4 \quad \vec{v} = c^2\frac{\vec{p}}{E}$$

Fuerza

$$\vec{f} = m\gamma\vec{a} + m\dot{\gamma}\vec{v}$$

$$f^\mu = ma^\mu = \frac{\mathrm{d}p^\mu}{\mathrm{d}\tau} = \gamma\left(cm\dot{\gamma}, \vec{f}\right) = \gamma\left(\frac{\vec{f}\cdot\vec{u}}{c}, \vec{f}\right)$$

III. ECS. ELECTRODINÁMICA CLÁSSICA

Continuidad

$$\frac{\partial \rho}{\partial t} + \vec{\nabla}(\rho \vec{u}) = 0 \qquad j^\mu = (c\rho, \rho\vec{u}) = (c\rho, \vec{\jmath})$$

$$\partial_\mu j^\mu = 0$$

Ecs. Maxwell

$$\vec{\nabla} \cdot \vec{B} = 0 \qquad \vec{\nabla} \times \vec{E} - \frac{1}{c}\partial_t \vec{B} = 0$$

$$\vec{\nabla} \cdot \vec{E} = 4\pi\rho \qquad \vec{\nabla} \times \vec{B} = \frac{4\pi}{c}\vec{\jmath} + \frac{1}{c}\partial_t \vec{E}$$

Potenciales:

$$\vec{B} = \vec{\nabla} \times \vec{A} \qquad \vec{E} = -\vec{\nabla}\phi - \frac{1}{c}\partial_t \vec{A}$$

Transformaciones gauge y gauge de Lorentz:

$$\phi' = \phi - \frac{1}{c}\partial_t \Lambda \qquad \vec{A}' = \vec{A} + \vec{\nabla}\Lambda$$

$$\frac{1}{c}\partial_t \phi + \vec{\nabla} \cdot \vec{A} = 0 \quad \rightarrow \quad \partial_\mu A^\mu = 0$$

$$\Box A^\mu = \frac{4\pi}{c} j^\mu \qquad \Box \equiv \partial_\mu \partial^\mu = \frac{\partial^2}{\partial t^2} - \nabla^2$$

Tensor de Faraday

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \quad \text{inv. gauge}$$

$$F^{0i} = -E^i = -F_{0i} \quad F^{ij} = -\epsilon^{ijk} B_k = F_{ij}$$

Tensor dual:

$$^*F^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}$$

Ecs. Maxwell covariantes:

$$\partial_\mu F^{\mu\nu} = \frac{4\pi}{c} j^\mu \qquad \partial_\mu {}^*F^{\mu\nu} = 0$$

Transformaciones $\vec{E}$, $\vec{B}$

$$\vec{E}'_\parallel = \vec{E}_\parallel \qquad \vec{B}'_\parallel = \vec{B}_\parallel$$

$$\vec{E}'_\perp = \gamma(\vec{E}_\perp + \vec{\beta} \times \vec{B}) \qquad \vec{B}'_\perp = \gamma(B_\perp - \vec{\beta} \times \vec{E})$$

Configuración estándar:

$$E'_x = E_x \qquad B'_x = B_x$$
$$E'_y = \gamma(E_y - \beta B_z) \qquad B'y = \gamma(B_y + \beta E_z)$$
$$E'_z = \gamma(E_z + \beta B_y) \qquad B'y = \gamma(B_z - \beta E_y)$$

Fuerza de Lorentz

$$\frac{\mathrm{d}p^\mu}{\mathrm{d}\tau} = f^\mu = \frac{q}{c} F^{\mu\nu} u_\mu = q\gamma \left(\frac{\vec{E} \cdot \vec{u}}{c}, \vec{E} + \frac{\vec{v}}{c} \times \vec{B} \right)$$

Invariantes

$$\vec{B}^2 - \vec{E}^2 \qquad \vec{E} \cdot \vec{B}$$

IV. RADIACIÓN

$$t' = t - \frac{R(t')}{c}$$

Potenciales de Liénard - Wiechert

$$\phi(\vec{x}, t) = \left. \frac{e}{(1 - \vec{\beta} \cdot \vec{n})R} \right|_{\tau = \tau_R} \qquad \vec{A}(\vec{x}, t) = \left. \frac{e\vec{\beta}}{(1 - \vec{\beta} \cdot \vec{n})R} \right|_{\tau = \tau_R}$$

Potencia Radiada:

$$\vec{S} = \frac{c}{4\pi}(\vec{E} \times \vec{B})$$

Fórmula de Larmor:

$$P = \frac{2}{3}\frac{e^2}{c^3}|\dot{\vec{v}}|^2$$

Fórmula de Larmor Relativista:

$$P = -\frac{2}{3}\frac{q^2}{m^2 c^3}\left(\frac{\mathrm{d}p^\mu}{\mathrm{d}\tau} \frac{\mathrm{d}p_\mu}{\mathrm{d}\tau} \right)$$

Distribución angular de potencia:

$$\frac{\mathrm{d}P}{\mathrm{d}\Omega} = \frac{e^2}{4\pi c} \frac{|\hat{n} \times [(\hat{n} - \vec{\beta}) \times \dot{\vec{\beta}}]|^2}{(1 - \hat{n} \cdot \vec{\beta})^5}$$