Formulari mecànica

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1. CINEMÀTICA

Defs.

$$\vec{v}=v\hat{v};\quad \vec{a}=\vec{a}_t+\vec{a}_n=\frac{\mathrm{d}v}{\mathrm{d}t}\hat{v}+v\frac{\mathrm{d}\hat{v}}{\mathrm{d}t};\qquad \vec{a}_n=\frac{\vec{v}^{\,2}}{R}\hat{u}_n$$

Parab:

$$\vec{a}=(0,-g);\quad \vec{v}=(v_{0x},v_{0y}-gt);\quad \vec{r}=(x_0+v_{0x}t,y_0+v_{0y}t-\frac{1}{2}gt^2)$$

Abast: $y=y_0$; $y_{max}=v_{0y}(t)=0$ o $y'(x)=0$.

MHS:

$$x(t)=A\cos(\omega t);\quad a(t)=-\omega^2\cdot x(t)$$

Curv:

$$\dot{\theta}=\frac{\mathrm{d}\theta}{\mathrm{d}t}=\omega;\quad \ddot{\theta}=\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2}=\alpha$$

Polars

$$\vec{r}=r\hat{u}_r;\quad \vec{v}=\dot{r}\hat{u}_r+r\dot{\theta}\hat{u}_\theta;\quad \vec{a}=(\ddot{r}-r\dot{\theta}^{\,2})\hat{u}_r+(2\dot{r}\dot{\theta}+r\ddot{\theta})\hat{u}_\theta$$

Mov. circular en polars:

$$\vec{r}:R\hat{u}_r;\quad \vec{v}=R\dot{\theta}\hat{u}_\theta;\quad \vec{a}=-R\omega^2\hat{u}_r+R\alpha\hat{u}_\theta$$

Descrip. vect Mov. Circ.:

$$\vec{a}_t=\vec{\alpha}\times\vec{r};\quad \vec{v}=\vec{\omega}\times\vec{r};\quad \vec{a}_n=\omega\times(\vec{\omega}\times\vec{r})$$

Mov. relat. transl.

$$\vec{r}\,'=\vec{r}-\vec{R};\quad \vec{v}\,'=\vec{v}-\vec{V};\quad \vec{a}\,'=\vec{a}-\vec{A};\quad \vec{A}=0\to\text{SRI}$$

Mov. relat. rot.

$$\vec{r}=\vec{r}\,';\quad \vec{v}=\vec{v}\,'+\vec{\omega}\times\vec{r};\quad \vec{a}=\vec{a}\,'+2\vec{\omega}\times\vec{v}\,'+\vec{\omega}\times(\vec{\omega}\times\vec{r})$$

Mov. rel. terra

$$a_{cf}=\omega^2 r\cos\lambda\quad \vec{a}_{co}=2\vec{\omega}\times\vec{v}';\quad g=g_0-\omega^2R\cos^2\theta$$

2. DINÀMICA

Defs.

$$\vec{F}=\frac{\mathrm{d}\vec{p}}{\mathrm{d}t};\quad \vec{p}\equiv m\vec{v}$$

$$\vec{\tau}=\vec{r}\times\vec{F};\quad \vec{\ell}=\vec{r}\times\vec{p};\qquad \vec{\tau}=\frac{\mathrm{d}\vec{\ell}}{\mathrm{d}t}$$

$$\vec{I}=\int_{t_0}^{t}\vec{F}(t')\mathrm{d}t';\quad \vec{I}=\Delta\vec{p}$$

Fict.

$$\vec{F}-m\vec{A}=m\vec{a}\to\vec{F}_{fict}=-m\vec{A}$$

3. TREBALL I ENERGIA

Defs.

$$\delta W=\vec{F}\cdot\mathrm{d}\vec{r}\to W_{1\to2}=\int_{\vec{r}_1}^{\vec{r}_2}\vec{F}\cdot\mathrm{d}\vec{r}\,';\quad W_{1\to2}=\Delta E_c$$

Força conservativa

$$\oint_\gamma \vec{F}\cdot\mathrm{d}\vec{r}=0;\ \vec{\nabla}\times\vec{F}=\vec{0};\quad \vec{F}=-\vec{\nabla}U;\quad W=-\Delta U;$$

$$\Delta E=\Delta(E_c+U)=0;\omega=\sqrt{\frac{K}{m}}=\sqrt{\frac{U''(x_0)}{m}}$$

Força no conservativa

$$W_{NC}=\Delta U+\Delta E_c=\Delta E=-\Delta U_{int};\quad \Delta E+\Delta U_{int}=0$$

4. SISTEMES DE PARTÍCULES

Centre de masses

$$M\vec{r}_{CM}=\sum_{i=1}^{N}m_i\vec{r}_i;\quad M\vec{r}_{CM}\int_{\mathcal{D}}\vec{r}\mathrm{d}m;$$

$$M=\sum_i m_i;\quad M=\int_{\mathcal{D}}\mathrm{d}m$$

2 partic.

$$\vec{r}_{CM}=\frac{m_1\vec{r}_1+m_2\vec{r}_2}{m_1+m_2}\equiv R;\quad \mu=\frac{m_1m_2}{m_1+m_2};\quad \vec{r}=\vec{r}_2-\vec{r}_1$$

$$E_c=\frac{1}{2}M\dot{\vec{R}}^{\,2}+\frac{1}{2}\mu\dot{\vec{r}}^{\,2}$$

$$\vec{F}_{ext}=0;\quad \vec{F}_{int}\neq0;\quad M\ddot{\vec{R}}=0;\quad \vec{F}=\mu\ddot{\vec{r}}$$

Energies

$$E_c=\frac{1}{2}M\vec{v}_{CM}^{\,2}+\frac{1}{2}\sum_i m_i(\vec{v}_i-\vec{v}_{CM})^2$$

Xocs

$$\sum_i\vec{F}_{ext}=0\to\vec{P}=\text{cte.}$$

Estudiat al CdM:

$$\vec{P}_1=-\vec{P}_2:\quad \vec{P}_1\,'=-\vec{P}_2\,'\to\vec{P}_{CM}=\vec{P}_{CM}\,'$$

- 1D:
  →Elàstic ($E_{c_{rel}}=$ cte.): $v_2'-v_1'=-(v_2-v_1)$

  $$v_1'=\frac{(m_1-m_2)v_1+2m_2v_2}{m_1+m_2};\quad v_2'=\frac{(m_2-m_1)v_2+2m_1v_1}{m_1+m_2}$$

  → Totalment intelàstic ($v_1'=v_2'=v_{CM}$):
  $v_2'-v_1'=-e(v_2-v_1)$

Sistemes massa variable:

$$mv=(m-\mathrm{d}m)(v+\mathrm{d}v)+\mathrm{d}mu$$

$$m\mathrm{d}v=-m(-v)\mathrm{d}m$$

$$v(t)-v(0)=-v_{expuls}\ln\frac{m(t)}{m(0)}$$

5.ROTACIONS i SÒLID RÍGID

Defs.

$$\vec{\tau} = \frac{\mathrm{d}\vec{\ell}}{\mathrm{d}t} \quad \text{respecte 0 i CM(SRNI)}$$

$$\vec{L} = \sum_i \vec{\ell}_i = \vec{r}_{CM} \times M\vec{v}_{CM} + \vec{L}_{CM}$$

Sòlid rígid: $|\vec{r}_i - \vec{r}_j| = \text{cte} \;\; \forall i, j$ Translació:

$$\vec{F}_{ext} = M \cdot \frac{\mathrm{d}\vec{v}_{Cm}}{\mathrm{d}t}$$

Rotació (pura)

$$\vec{L} = \sum_i \vec{\ell}_i = \sum_i m_i \vec{r}_i \times (\vec{\omega} \times \vec{r}_i) = \mathcal{I} \cdot \vec{\omega}$$

$$L_z = I\omega$$

Moment d'inèrcia

$$I = \sum_i m_i \rho_i^2; \quad I = \int \mathrm{d}I = \int_{\mathcal{D}} \rho^2 \mathrm{d}m; \quad \rho_i = r_i \sin\theta_i$$

Teorema d'Steiner

$$I = I_{CM} + Mh^2; \qquad h \equiv \text{dist. entre els 2 eixos}$$

Energia cinètica

$$E_c = \frac{1}{2} M v_{CM}^2 + \frac{1}{2} I\omega^2$$

$$\frac{1}{2} I\omega^2 = \frac{1}{2} \frac{L^2}{I}$$

Dinàmica sòlid rígid:

$$\sum_i \vec{F}_i = M\vec{a}_{CM}; \qquad \sum_i \vec{\tau}_i = I\vec{\alpha}$$

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6. GRAVITACIÓ

Força, camp, potencial i energia potencial

$$\vec{F}_{12} = -G\frac{m_1 m_2}{r_{12}^2}\hat{r}_{12}; \quad \vec{F} = -Gm_0 \sum_{i=1}^{N} \frac{m_i}{r_{i0}^2}\hat{r}_{i0}$$

$$\vec{g} = \frac{\vec{F}}{m_0} = -G\sum_i \frac{m_i}{r_{i0}^2}\hat{r}_{i0}; \quad \vec{g} = -g\int_{\mathcal{D}} \frac{\mathrm{d}m}{r'^2}\hat{r}' \quad r' = \mathrm{d}(\mathrm{d}m, P)$$

$$W = -G\int_{r_{10}}^{\infty} \frac{m_1 m_0}{r_{10}^2}\mathrm{d}r_{10} - G\int_{r_{20}}^{\infty} \frac{m_2 m_0}{r_{20}^2}\mathrm{d}r_{20} - \ldots = -Gm_0\sum_{i=1}^{N} \frac{m_i}{r_{i0}} = U(P)$$

$$V(P) = \frac{U(P)}{m} \qquad V = -G\int_{\mathcal{D}} \frac{\mathrm{d}m}{r'}$$

$$\vec{F} = -\vec{\nabla}U \qquad \vec{g} = -\vec{\nabla}V$$

Teorema de Gauss

$$\phi = \oint_{\mathcal{S}} \vec{g} \cdot \mathrm{d}\vec{S} = -4\pi G M_{int}$$
