Formulari d'òptica

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I. ÒPTICA ONDULATÒRIA

Equacions de Maxwell:

$$\vec{\nabla} \times \vec{H} = \frac{4\pi}{c}\vec{\jmath} + \frac{1}{c}\frac{\partial \vec{D}}{\partial t} \qquad \vec{\nabla} \cdot \vec{D} = 4\pi\rho$$

$$\vec{\nabla} \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t} \qquad \vec{\nabla} \cdot \vec{H} = 0$$

$$\vec{D} = \varepsilon\vec{E} \qquad \vec{B} = \mu\vec{H} \qquad \vec{\jmath} = \sigma\vec{E}$$

Diel. hom. i isòtr. descarr.: $\sigma = 0,\ \mu = \text{cte},\ \varepsilon = \text{cte},\ \rho = 0$

$$\nabla^2\vec{E} = \frac{\varepsilon\mu}{c^2}\frac{\partial^2\vec{E}}{\partial t^2} \qquad \nabla^2\vec{H} = \frac{\varepsilon\mu}{c^2}\frac{\partial^2\vec{H}}{\partial t^2}$$

$$v = \frac{c}{\varepsilon\mu} \qquad n = \sqrt{\varepsilon\mu} \qquad n = \frac{c}{v}$$

$$\vec{E} = \vec{E}_0 e^{i(\omega t - k\vec{r}\hat{s})} \qquad \vec{H} = \vec{H}_0 e^{i(\omega t - k\vec{r}\hat{s})}$$

$$\vec{E} = \frac{\vec{E}_0}{r} e^{i(\omega t - kr)}$$

$$k = \frac{2\pi}{\lambda} \quad v = \frac{\lambda}{T} \quad v = \frac{\omega}{k} \quad \omega = 2\pi\nu \quad p = \frac{\omega}{c} \quad d = \vec{r}\cdot\hat{s}$$

$$\vec{H} \times \hat{s} = n\vec{E} \qquad \hat{s} \times \vec{E} = \frac{1}{n}\vec{H} \qquad \hat{s}\cdot\vec{E} = 0 \qquad \hat{s}\cdot\vec{H} = 0$$

Vector de Poynting i Energia:

$$\vec{S} = \frac{c}{4\pi}(\vec{E} \times \vec{H}) \qquad \frac{\partial U}{\partial t} = -\int_{\mathcal{S}} \vec{S}\,\mathrm{d}\vec{\mathcal{S}}$$

$$|\vec{S}| = \frac{cn}{4\pi}E^2 = \frac{cn}{4\pi}E_0^2\cos^2(\omega t + \varphi)$$

$$I = \langle S \rangle = \frac{1}{\tau}\int_0^\tau |\vec{S}|\mathrm{d}t \qquad I = \frac{cn}{8\pi}E_0^2$$

Polarització:

$$\left.\begin{aligned} E_x &= A_1\cos(\omega t) \\ E_y &= A_2\cos(\omega t + \delta)\end{aligned}\right\} \rightarrow \quad \frac{E_x^2}{A_1^2} + \frac{E_y^2}{A_2^2} - \frac{2E_xE_y}{A_1A_2}\cos\delta = \sin^2\delta$$

$$\tan(2\xi) = \frac{2A_1A_2\cos\delta}{A_1^2 - A_2^2} \qquad \begin{cases} 0 < \delta < \pi \rightarrow \text{dextrògira} \\ \pi < \delta < 2\pi \rightarrow \text{levògira}\end{cases}$$

Llum polaritzada circular: $A_x = A_y$ i $\delta = \frac{\pi}{2}$ o $\delta = \frac{3\pi}{2}$

$$I = \frac{cn}{8\pi}(A_1^2 + A_2^2)$$

Contrast òptic i paràmetres d'Stokes:

$$C = G = \frac{I_M - I_m}{I_M + I_m}$$

$$I' = A_1^2 + A_2^2, \quad M' = A_1^2 - A_2^2,$$
$$C' = 2A_1A_2\cos\delta, \quad S' = 2A_1A_2\sin\delta$$

II. PROPAGACIÓ DE LA LLUM EN MEDIS

Conservació de la component tangencial:

$$\hat{s} = (\alpha,\beta,\gamma) \quad \hat{s}' = (\alpha',\beta',\gamma') \quad \hat{s}''(\alpha'',\beta'',\gamma'')$$

$$\hat{s} = (\sin\varphi, 0, \cos\varphi) \quad \hat{s}' = (\sin\varphi', 0, \cos\varphi') \quad \hat{s}'' = (\sin\varphi'', 0, -\cos\varphi'')$$

$$\vec{E}_t + \vec{E}_t'' = \vec{E}_t' \quad \rightarrow \quad \begin{cases} E_x + E_x'' = E_x' \\ E_y + E_y'' = E_y' \end{cases} \qquad \forall t, x, y$$

$$\begin{aligned} E_y &= A_y e^{ip[ct - n(\alpha x + \beta y + \gamma z)]} \\ E_y' &= A_y' e^{ip'[ct - n(\alpha' x + \beta' y + \gamma' z)]} \\ E_y'' &= A_y'' e^{ip''[ct - n(\alpha'' x + \beta'' y + \gamma'' z)]} \end{aligned}$$

1. $\forall t$

$$p = p' = p'' \qquad \nu = \nu' = \nu'' \qquad \lambda = \lambda'' \neq \lambda'$$

2. $\forall y$

$$n\beta = n'\beta' = n\beta''$$

$$\text{si } \beta = 0 \ \rightarrow\ \beta = \beta' = \beta'' \ \rightarrow\ \hat{s}, \hat{s}', \hat{s}'' \text{ coplanaris.}$$

3. $\forall x$:

$$n\alpha = n'\alpha' = n\alpha''$$

$$\alpha'' = \alpha \ \rightarrow\ \sin\varphi = \sin\varphi'' \ \rightarrow\ \varphi'' = \varphi \ \rightarrow\ \text{REFLEXIÓ}$$
$$n\alpha = n'\alpha' \ \rightarrow\ n\sin\varphi = n'\sin\varphi' \ \rightarrow\ \text{SNELL}$$

Equacions de Fresnel:

$$t_\parallel = \frac{A_\parallel'}{A_\parallel} = \frac{2\sin\varphi'\cos\varphi}{\sin(\varphi'+\varphi)\cos(\varphi'-\varphi)} \qquad r_\parallel = \frac{A_\parallel''}{A_\parallel} = \frac{\tan(\varphi'-\varphi)}{\tan(\varphi'+\varphi)}$$

$$t_\perp = \frac{A_\perp'}{A_\perp} = \frac{2\sin\varphi'\cos\varphi}{\sin(\varphi'+\varphi)} \qquad r_\perp = \frac{A_\perp''}{A_\perp} = \frac{\sin(\varphi'-\varphi)}{\sin(\varphi'+\varphi)}$$

Incidència normal: $\varphi = 0$

$$t_\parallel = t_\perp = \frac{2n}{n+n'} \qquad r_\parallel = r_\perp = \frac{n-n'}{n+n'}$$

Angle de Brewster: $\varphi + \varphi' = \pi/2 \quad \rightarrow \quad A_\parallel'' = 0$

$$\tan\varphi_B = \frac{n'}{n}$$

Signes:

$$\rightarrow \text{ trans.} : +$$

$$\rightarrow \text{ refl.} \begin{cases} n < n' \begin{cases} \parallel: \begin{cases} \varphi < \varphi_B : - \\ \varphi > \varphi_B : + \end{cases} & \text{CF} : \pi \\ \perp: - \end{cases} \\ n > n' \begin{cases} \parallel: \begin{cases} \varphi < \varphi_B : + \\ \varphi > \varphi_B : - \end{cases} & \text{CF} : 0 \\ \perp: - \end{cases} \end{cases}$$

Conservació de l'Energia:

$$R_{\parallel} = r_{\parallel}^2 \qquad T_{\parallel} = \frac{n' \cos\varphi'}{n \cos\varphi} t_{\parallel}^2$$

$$R_{\parallel} + T_{\parallel} = 1$$

Reflexió total: $n > n'$

$$\sin\varphi_\ell = \frac{n'}{n} \equiv N \qquad \sin\varphi' = \frac{1}{N}\sin\varphi$$

$$\varphi > \varphi_\ell \ \rightarrow \ \sin\varphi' > 1 \ \rightarrow \ \cos\varphi' = -\frac{i}{N}\sqrt{\sin^2\varphi - N^2}$$

$$A''_{\parallel} = A_{\parallel} \mathrm{e}^{i2\alpha} \qquad A''_{\perp} = A_{\perp} \mathrm{e}^{2i\beta} \qquad \delta = 2(\alpha - \beta)$$

$$\tan\alpha = \frac{\sqrt{\sin^2\varphi - N^2}}{N^2 \cos\varphi} \qquad \tan\beta = \frac{\sqrt{\sin^2\varphi - N^2}}{\cos\varphi}$$

Refr: ona evanescent.

$$\vec{E}'_{\mathbb{R}} = \vec{A}' \mathrm{e}^{-\kappa z} \cos\omega\left(t - \frac{x\sin\varphi}{v}\right)$$

Medis conductors

$$\tilde{\varepsilon} = \varepsilon - i\frac{4\pi\sigma}{\omega} \qquad \tilde{n} = n - i\chi \ \ n^2 - \chi^2 = \varepsilon \ \ n\chi = \sigma T$$

Drude: $n \approx \xi \approx \sqrt{\sigma T}$

Refr:

$$\vec{E} = \vec{E}_0 \mathrm{e}^{-\frac{\omega}{c}\chi \vec{r}\cdot\hat{s}} \mathrm{e}^{i\omega\left(t - \frac{n}{c}\vec{r}\cdot\hat{s}\right)}$$

Refl:

$$R = \left|\frac{1-\tilde{n}}{1+\tilde{n}}\right|^2$$

Amplituds:

$$\frac{A''_{\perp}}{A''_{\parallel}} = M\mathrm{e}^{i\delta}$$

Medis anisòtrops:

$$n = \sqrt{\varepsilon} \quad \vec{D} = \varepsilon\vec{E} \qquad \vec{\nabla}\times\vec{H} = \frac{1}{c}\frac{\partial\vec{D}}{\partial t} \qquad \vec{\nabla}\times\vec{E} = -\frac{1}{c}\frac{\partial\vec{H}}{\partial t}$$

$$n(\vec{H}\times\hat{s}) = \vec{D} \ \ n(\hat{s}\times\vec{E}) = \vec{H} \ \ \vec{H}\cdot\hat{s} = 0 \ \ \vec{D}\cdot\hat{s} = 0$$

$$\vec{D} = n^2(\vec{E} - \hat{s}(\vec{E}\cdot\hat{s})) \quad n = \frac{c}{v_n}$$

$$v_i = \frac{c}{\sqrt{\varepsilon_i}} \qquad D_i = \frac{c^2\vec{E}\cdot\hat{s}}{v_i - v_n^2} s_i \qquad \vec{D}\cdot\hat{s} = 0$$

$$\frac{s_x^2}{v_x^2 - v_n^2} + \frac{s_y^2}{v_y^2 - v_n^2} + \frac{s_z^2}{v_z^2 - v_n^2} = 0 \quad \rightarrow \begin{cases} v_{n1} \\ v_{n2} \end{cases} \qquad \vec{D}_1\cdot\vec{D}_2 = 0$$

III.INTERFERÈNCIES

Experiment de Young:

$$I = A_1^2 + A_2^2 + 2A_1A_2\cos\delta \ \ \rightarrow \ \ A_1 = A_2 \rightarrow \ \ I = 4A^2\cos^2\delta/2$$

$$\delta = \frac{2\pi}{\lambda}\Delta \qquad \Delta = S_2 - S_1 \qquad \Delta = \frac{xd}{D} \quad xd \ll D$$

Max : $\cos^2\delta/2 = \pi k$ Min: $\cos^2(\delta/2) = (2k+1)\pi/2$

Interfranja: $\Delta x = \lambda D/d$ Biprisma Fresnel: $d = 2\alpha(n-1)a$

Mirall Lloyd:

$$\delta = (2k+1)\pi \quad \Delta = (2k+1)\frac{\lambda}{2} \quad x_k = (2k+1)\frac{\lambda D}{2d}$$

Interferències en làmines planoparal·leles

$$\Delta = 2nd\cos\varepsilon'$$

Trans:

$$I_t = \frac{a^2}{1 + \frac{4r^2}{(1-r^2)^2}\sin^2\frac{\delta}{2}} \quad \text{Max:}\frac{\delta}{2} = k\pi \quad \text{Min}: \delta = (2k+1)\pi$$

Refl: desf π

$$I = \frac{a^2\sin\frac{\delta}{2}}{\frac{(1-r^2)^2}{4r^2} + \sin^2\frac{\delta}{2}} \quad \text{Min:}\frac{\delta}{2} = k\pi \quad \text{Max}: \delta = (2k+1)\pi$$

Anells: numerar per fora

$$\text{Max}: 2nd\cos\varepsilon' = k\lambda \quad \text{Min}: 2nd\cos\varepsilon' = (2k+1)\frac{\lambda}{2}$$

$$k_{max} = \frac{2nd}{\lambda}$$

Fabry-Pérot: poder resol. $|\lambda/\Delta\lambda| = (\pi k r)/(1-r^2)$

Anells Newton:

$$d = k\frac{\lambda}{2} \qquad \frac{r_k}{d_k} = \frac{2R - d_k}{r_k} \qquad 2d_k + \frac{\lambda}{2} = (2k+1)\frac{\lambda}{2} \quad R = \frac{r_k^2}{k\lambda}$$

Michelson:

$$d = \ell_2 - \ell_1 \quad \Delta = 2d \quad Max: 2d = k\lambda \quad \rho = f'\tan\varepsilon$$

IV. DIFRACCIÓ

$$I_p = C16a^2b^2\left(\frac{\sin k\alpha' a}{k\alpha' a}\right)^2\left(\frac{\sin k\beta' b}{k\beta' b}\right)^2$$

$$I_p = C(\pi R^2)^2\left(\frac{2J_1(z)}{z}\right)^2 \qquad z \equiv \frac{kR\rho'}{f'} \qquad \rho' = 1,22\frac{\lambda}{\Phi}f'$$

$$I_p = C\frac{\sin^2(Nk\alpha' d)}{\sin^2 k\alpha' d}\left(\frac{\sin k\alpha' a}{k\alpha' a}\right)^2$$

$$\alpha' = \frac{x'}{f'} \quad \beta' = \frac{y'}{f'} \quad x' = a \quad y' = b$$

$$2a\cdot a' = \lambda f' \qquad \Delta x'\cdot 2d = \lambda f' \qquad \rightarrow \qquad \Delta x' N d = \lambda f'$$

$$n.fr. = \mu = \frac{2a'}{\Delta x'} \quad \mu\Delta x' = \Delta M \qquad N\ min \ \ N-1\ max.$$

$$\tan\alpha = \Delta x'/D \qquad \tan\alpha = \rho/f$$

$$\frac{\lambda}{\delta\lambda} = mN$$