

FORMULARI DE FÍSICA QUÀNTICA

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CONSTANTS

$$h = 6,62608 \times 10^{-34} \text{ J}\cdot\text{s} \quad \hbar = 1,05459 \times 10^{-34} \text{ J}\cdot\text{s}$$

$$e = 1,60219 \times 10^{-19} \text{ C}$$

$$k = 1,38066 \times 10^{-23} \text{ J}\cdot\text{K}^{-1}$$

$$\sigma = 5,67051 \times 10^{-8} \text{ W}\cdot\text{m}^{-2}\cdot\text{K}^{-4}$$

$$m_e = 9,10953 \times 10^{-31} \text{ kg} \quad m_p = 1,67265 \times 10^{-27} \text{ kg}$$

$$m_p = 1,67265 \times 10^{-27} \text{ kg} \quad m_n = 1,67493 \times 10^{-27} \text{ kg}$$

$$R_H = 1,09678 \times 10^7 \text{ m}^{-1} \quad (\text{experimental})$$

$$R_\infty = \frac{m_e}{4\pi\hbar^3 c} \left(\frac{e^2}{4\pi\epsilon_0} \right)^2 = 1,09737 \times 10^7 \text{ m}^{-1}$$

$$E_1(H) = -13,6 \text{ eV}$$

$$a_0 = \frac{4\pi\epsilon_0 \hbar^2}{m_e e^2} = 5,29177 \times 10^{-11} \text{ m}$$

$$c = 3 \times 10^8 \text{ m/s}$$

$$m_e c^2 = 0,511 \text{ MeV} \quad m_p c^2 = 938 \text{ MeV} \quad m_n c^2 = 939 \text{ MeV}$$

RADIACIÓ

$$R_\nu = I_\nu a_\nu = E_{\text{abs}} \quad (eq. \text{termic}) \quad a + r + t = 1$$

$$\text{Llei de Stefan-Boltzman: } R_T = \sigma T^4 \quad [Wm^{-2} \equiv J \cdot s^{-1} m^{-2}]$$

$$\text{Llei de Planck: } \rho(\nu, T) = \left(\frac{8\pi\nu^2}{c^3} \right) \frac{h\nu}{e^{h\nu/k_B T} - 1}$$

$$\rho(\lambda, T) = \left(\frac{8\pi ch}{\lambda^5} \right) \frac{1}{e^{hc/\lambda k_B T} - 1} \quad R_T = \int_0^\infty R_T(\nu) d\nu \quad R_T(\nu) = \frac{c}{4} \rho_T(\nu)$$

$$\text{Llei del desplaçament de Wien: } \rho_T(\nu) = AT^3 \quad \rho_T(\lambda) = BT^5$$

$$X_M = \nu_M T^{-1}$$

$$Z_M = \lambda_M T = 2,898 \times 10^{-3} \text{ mK}$$

EFFECTE FOTOELÈCTRIC

$$\text{Flux energètic: } \phi_d = \frac{P}{4\pi d^2} \quad \text{Potencial de frenada: } V_f = \frac{h}{e} \nu - \frac{W_0}{e}$$

$$\text{Energia de salt: } \frac{1}{2} m_e v_{\text{max}}^2 = h\nu - W_0 = eV_0 \quad \text{Freqüència llindar: } \nu_0 = \frac{W_0}{h}$$

EFFECTE COMPTON

$$\lambda_1 - \lambda_0 = \frac{h}{mc} (1 - \cos\theta) \quad \lambda_C = \frac{h}{m_e c} = 0,02426 \text{ \AA} \quad \text{Canvi relatiu: } \left| \frac{\Delta\lambda}{\lambda_0} \right|$$

$$E_{\text{foto}} = h\nu \quad P_{\text{foto}} = \frac{h\nu}{c} \quad \text{Creació de parells: } h\nu = m_{e^+} + T_+ + m_{e^-} + T_-$$

MODELS ATÒMICS

$$K = \frac{1}{\lambda} = \frac{\nu}{c} = R_\infty Z^2 \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right) \quad \forall n_i > n_f$$

$$\text{Radi de Bohr: } r_n = \frac{4\pi\epsilon_0 \hbar^2 n^2}{Z\mu e^2}$$

$$E_n = -\frac{1}{2} \frac{1}{4\pi\epsilon_0} \frac{Ze^2}{r_n} = -\frac{\mu}{2\hbar^2} \left(\frac{Ze^2}{4\pi\epsilon_0} \right)^2 \frac{1}{n^2} \quad \nu = \frac{n\hbar}{mr} = \frac{1}{4\pi\epsilon_0} \frac{Ze^2}{n\hbar}$$

$$L = \mu vr = n\hbar \quad R_M = R_\infty \frac{\mu}{m} \quad R_{MH} = 10968100 \text{ m}^{-1}$$

$$\mu = \frac{mM}{m+M} \quad (\text{Normalment: } \mu \approx m_e) \quad \frac{\lambda_1}{\lambda_2} = \frac{\mu_2}{\mu_1}$$

Resolució de l'àtom de Bohr:

$$\left\{ \begin{array}{l} \frac{\mu v^2}{r} = F(r) = -V'(r) \\ \mu vr = n\hbar \end{array} \right\} \rightarrow E_n = E_{\text{cin}} + E_{\text{pot}} = \frac{1}{2} \mu v^2 + V$$

$n_f = 1$	Lyman
$n_f = 2$	Balmer
$n_f = 3$	Paschen
$n_f = 4$	Bracket
$n_f = 5$	Pfund

$$V_{\text{Gravitatori}} = -\frac{GmM}{r} \quad V_{\text{Electric}} = -\frac{1}{4\pi\epsilon_0} \frac{Ze^2}{r}$$

$$\text{Llei de Moseley: } \frac{1}{\lambda} = R_H (Z-a)^2 \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right) \rightarrow \frac{1}{\lambda_\alpha} = R_H (Z-1)^2 \left(1 - \frac{1}{2^2} \right)$$

PROPIETATS ONDULATÒRIES I INDETERMINACIÓ DE HEISENBERG

$$\text{Hipòtesi de De Broglie: } \lambda = \frac{h}{P} \quad \nu = \frac{E}{h}$$

$$\text{Correcció relativista: } \sqrt{m^2 c^4 + c^2 |\vec{p}|^2} = mc^2 + T \quad \text{amb } T = eV$$

$$\text{Si } T \ll mc^2 \Rightarrow T = \frac{1}{2} mv^2 \quad P = mv \quad \nu = \frac{c}{\lambda}$$

$$\text{Incerteses: } \Delta x \Delta P \geq \hbar \quad \Delta E \Delta t \geq \hbar \quad \Delta L \Delta \phi \geq \hbar \quad \text{Amplada natural: } \Delta \nu = \Delta E/h$$

EQ. DE SHRÖDINGER, FUNCIONS DE PROBABILITAT I VALORS ESPERATS

Shrödinger (DT):

$$i\hbar \frac{\partial}{\partial t} \psi(x,t) = -\frac{\hbar^2}{2\mu} \frac{\partial^2}{\partial x^2} \psi(x,t) + V(x)\psi(x,t) = E\psi(x,t)$$

$$-i\hbar \frac{\partial}{\partial t} \psi^*(x,t) = -\frac{\hbar^2}{2\mu} \frac{\partial^2}{\partial x^2} \psi^*(x,t) + V(x)\psi^*(x,t) = E\psi^*(x,t)$$

$$\text{Shrödinger (IDT): } -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \psi(x) + V(x)\psi(x) = E\psi(x)$$

$$\int_{-\infty}^{\infty} \psi(x,t)\psi^*(x,t) dx = \int_{-\infty}^{\infty} |\psi(x,t)|^2 dx = 1 \quad a_n = \int_{-\infty}^{\infty} \phi_n^*(x)\psi(x,0) dx$$

$$\psi(x,t) = \sum_{n=1}^{\infty} a_n \phi_n(x) e^{iE_n t/\hbar} \quad \sum_n |a_n|^2 = 1 \quad P(E_n) = |a_n|^2 = \int_{-\infty}^{\infty} \phi_n^*(x)\phi_n(x) dx$$

$$\text{Operadors: } E^{op} = i\hbar \frac{\partial}{\partial t} \quad P^{op} = -i\hbar \frac{\partial}{\partial x} \quad H^{op} = \frac{(P^{op})^2}{2m} + V(x,t)$$

$$\text{Valors esperats: } \langle E^m \rangle = \int_{-\infty}^{\infty} \psi^*(x,t) (E^{op})^m \psi(x,t) dx = \sum_n |a_n|^2 (E_n)^m$$

$$\langle f(E) \rangle = \sum_n |a_n|^2 f(E_n) \quad \Delta f = \sqrt{\langle f^2 \rangle - \langle f \rangle^2}$$

$$H\psi = E_n \psi \quad P^{op}\psi(x,t) = P\psi(x,t) \quad \langle P^2 \rangle = 2m \langle E \rangle$$

$$\text{Teorema d'Ehrenfest: } \frac{d\langle x \rangle}{dt} = \frac{\langle P \rangle}{m}; \quad \frac{d\langle P \rangle}{dt} = -\left\langle \frac{\partial V}{\partial x} \right\rangle$$

BARRERA DE POTENCIAL (amplada a)

$$E \leq V_0 \Rightarrow k^2 = \frac{2m(V_0 - E)}{\hbar^2} = -k'^2 \Leftrightarrow E \geq V_0 \quad k = \left[\frac{2mE}{\hbar^2} \right]^{1/2}$$

$$R = \left[1 + \frac{4E(V_0 - E)}{V_0^2 \sinh^2(k_1 a)} \right]^{-1} \quad T = \left[1 + \frac{V_0^2 \sinh^2(k_1 a)}{4E(V_0 - E)} \right]^{-1}$$

POU DE PARETS INFINITES (amplada L=2a)

$$\phi_n(x) = \begin{cases} \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right) & x \in [0, L] \\ 0 & x \notin [0, L] \end{cases} \quad k = \left[\frac{2m(E - V_0)}{\hbar^2} \right]^{1/2}$$

$$\phi_n(x) = \begin{cases} \sqrt{\frac{1}{a}} \sin\left(\frac{n\pi(x+a)}{2a}\right) & x \in [-a, a] \\ 0 & x \notin [-a, a] \end{cases} \quad E_n - V_0 = \frac{\hbar^2 \pi^2}{2mL^2} n^2$$

POU QUADRAT (amplada L=2a)

$$\alpha = \left[\frac{2m(V_0 - E)}{\hbar^2} \right]^{1/2} \quad k = \left[\frac{2mE}{\hbar^2} \right]^{1/2} \quad \beta = \left[\frac{2m(V_0 + E)}{\hbar^2} \right]^{1/2}$$

$$\left(\frac{\alpha a}{\xi} \right)^2 + \left(\frac{k a}{\eta} \right)^2 = \theta_0^2 = \left(\frac{2mV_0 a^2}{\hbar^2} \right) \quad \text{Parells } \tan \xi = \frac{ka}{\alpha a}$$

$$\text{Senars } \text{ctg} \xi = -\frac{ka}{\alpha a}$$

$$0 \leq \theta_0 \leq \frac{\pi}{2} \Rightarrow 1 \text{ solució} \quad R = \left[1 + \frac{4E(V_0 + E)}{V_0^2 \sin^2(\beta L)} \right]^{-1}$$

$$\frac{\pi}{2} \leq \theta_0 \leq \pi \Rightarrow 2 \text{ solucions} \quad T = \left[1 + \frac{V_0^2 \sin^2(\beta L)}{4E(V_0 + E)} \right]^{-1}$$

$$\pi \leq \theta_0 \leq \frac{3\pi}{2} \Rightarrow 3 \text{ solucions}$$

OSCIL·LADOR HARMÒNIC

$$\underbrace{\frac{\hbar}{2m} \frac{d^2}{dx^2}}_{(p^{op})} \phi_n + \underbrace{\frac{1}{2} m \omega^2 x^2}_{V(x)} \phi_n = E_n \phi_n \quad E_n = \left(n + \frac{1}{2} \right) \hbar \omega \quad E_{clas} = \frac{1}{2} m \omega^2 A^2$$

$$\alpha \equiv \left(\frac{2m\omega}{\hbar} \right)^{1/2} \quad \xi \equiv \alpha x \quad \phi_n(x) = \left(\frac{\sqrt{\pi}}{\alpha} 2^n n! \right)^{-1/2} e^{-\frac{1}{2}\xi^2} H_n(\xi)$$

$$\langle x \rangle = \int_{-\infty}^{\infty} \phi_n^*(x) x \phi_n(x) dx = 0 \quad \int_{-\infty}^{\infty} \phi_n^*(x) \phi_m(x) dx = \delta_{nm}$$

$$\int_{-\infty}^{\infty} H_n(\xi) H_m(\xi) e^{-\xi^2} d\xi = 2^n n! \sqrt{\pi} \delta_{nm}$$

ÀTOMS AMB UN SOL ELECTRÓ

$$n = 1, 2, 3, \dots \quad l = 0, 1, 2, \dots, n-1 \quad m = -l, -l+1, \dots, 0, \dots, l$$

$$\text{Harmònics esfèrics: } Y_{l,m}(\theta, \varphi) = (-1)^m \sqrt{\frac{2l+1}{4\pi} \frac{(l-m)!}{(l+m)!}} e^{im\varphi} P_l^m(\cos\theta)$$

$$\text{Polinomis de Legendre: } P_l^m(z) = \frac{(-1)^m}{2^l l!} (1-z^2)^{m/2} \frac{d^{l+m}}{dz^{l+m}} (z^2-1)^l$$

$$\Psi(\vec{r}) = R_{n,l}(r) Y_{l,m}(\theta, \varphi) \quad T(t) = e^{-i\frac{E_n t}{\hbar}} \int_0^\infty |R_{n,l}(r)|^2 dr = 1$$

$$\int_\Omega Y_{l,m}^*(\theta, \varphi) Y_{l,m}(\theta, \varphi) d\Omega = \delta_{l'l} \delta_{m'm} \quad P = \int_{R_1}^{R_2} r^2 |R_{n,l}(r)|^2 dr \int_\Omega Y^* Y d\Omega$$

$$R_{n,l}(r) = -\sqrt{\left(\frac{2z}{na_0} \right)^3 \frac{(n-l-1)!}{2n}} e^{-\frac{zr}{na_0}} \left(\frac{2r}{na_0} \right)^l L_{n+l}^{2l+1} \left(\frac{2r}{na_0} \right)$$

$$R_{1,0} = 2 \left(\frac{z}{a_0} \right)^{3/2} e^{-\frac{zr}{a_0}} \quad R_{2,0} = \left(\frac{z}{2a_0} \right)^{3/2} \left(2 - \frac{zr}{a_0} \right) e^{-\frac{zr}{2a_0}}$$

$$R_{2,1} = \left(\frac{z}{2a_0} \right)^{3/2} \cdot z \cdot r \frac{1}{a_0 \sqrt{3}} \left(2 - \frac{zr}{a_0} \right) e^{-\frac{zr}{2a_0}}$$

$$\langle r \rangle = \int_0^\infty r P_{n,l}(r) dr = \int_0^\infty R_{n,l}^* R_{n,l} r dr \quad \langle L_x \rangle = \langle L_y \rangle = 0$$

$$L_\pm = L_x \pm iL_y \quad L_\pm Y_{l,m} = \sqrt{l(l+1) - m(m \pm 1)} \hbar Y_{l,m \pm 1}$$

$$L_x = (L_+ + L_-)/2 \quad L_y = (L_+ - L_-)/(2i)$$

$$\langle L_z \rangle = \sum m_l \hbar |C_{n,l,m}|^2 \quad \langle L^2 \rangle = \sum l(l+1) \hbar^2 |C_{n,l,m}|^2 \quad \langle V(r) \rangle = -\frac{e^2}{4\pi\epsilon_0} \left\langle \frac{1}{r} \right\rangle$$

ANNEX (integrals més usuals)

$$\int_{-\infty}^{\infty} e^{-ax^2} dx = \sqrt{\frac{\pi}{a}} \quad \int_{-\infty}^{\infty} e^{-ax^2+bx+c} dx = \sqrt{\frac{\pi}{a}} e^{\frac{b^2}{4a}+c} \quad \int_0^\infty x^n e^{-ax} dx = \frac{\Gamma(n+1)}{a^{n+1}}$$

$$\int_0^\infty \frac{x^3}{e^x - 1} dx = \frac{\pi^4}{15} \quad \int_{-\infty}^{\infty} x^2 e^{-ax^2} dx = \frac{\sqrt{\pi}}{2a^{3/2}}$$