

Física cuántica

Formulario

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Ley Stefan:

$$R_T = \sigma T^4 = \frac{c}{4} \rho_T = \int_0^\infty \rho_T(\nu) d\nu$$

Ley Plank:

$$\rho_T(\nu) = \frac{8\pi\nu^2}{c^3} \frac{h\nu}{e^{\frac{h\nu}{kT}} - 1}, \quad \rho_T(\lambda) = \frac{8\pi hc}{\lambda} \frac{1}{e^{\frac{hc}{\lambda kT}} - 1}$$

Fotoeléctrico:

$$T = h\nu - \phi \rightarrow \nu_0 = \phi/h, \text{ corte } V_0 = \frac{h}{e}\nu - \frac{\phi}{e}$$

Compton:

$$\lambda' = \lambda + \frac{h}{mc}(1 - \cos\theta)$$

$$K = \frac{(hc/\lambda)^2}{\frac{hc}{\lambda} + \frac{mc^2}{1 - \cos\theta}}, \quad \cotg\varphi = \left(1 + \frac{h}{m\lambda c}\right) \tg\frac{\theta}{2}$$

Rutherford:

$$\tg\frac{\beta}{2} = \frac{KqQ}{2bE}, \quad dn = Nb db d\varphi = N\sigma(\beta)d\Omega$$

Secc. eff.:

$$\sigma(\beta) = \left(\frac{KqQ}{4E}\right)^2 \sin^{-4}(\beta/2)$$

$$L = mbv_0, \quad r_{\min} = K\frac{qQ}{E}, \quad b = \frac{KqQ}{2E} \sqrt{\frac{m}{\mu}} \cotg\frac{\beta}{2}$$

Bohr:

$$r = \frac{L^2}{\mu Z e^2} = \frac{(n\hbar)^2}{mZ e^2} = \frac{n^2 a_0}{Z},$$

$$a_0 = \hbar^2/mc^2 = 0,529 \text{ \AA}$$

$$E_n = -\frac{\mu Z^2 e^4}{2n^2 \hbar^2} = Z^2 \frac{E_1}{n^2}, \quad E_1 = -\frac{me^4}{2\hbar^2} = -13,6$$

$$\nu_{n,m} = \frac{mZ^2 e^4}{2h\hbar^2} \left(\frac{1}{m^2} - \frac{1}{n^2}\right), \quad \frac{1}{\lambda} = Z^2 R \left(\frac{1}{m^2} - \frac{1}{n^2}\right)$$

$$\text{Rydberg } R = \frac{2\pi^2 m e^4}{ch^3} = 109677,576 \text{ cm}^{-1}$$

Franck-Hertz: Hg 4,9eV, $\lambda = 2536 \text{ \AA}$

Moseley (r. X):

$$\frac{1}{\lambda_{K\alpha}} = R(Z-1)^2 \left(\frac{1}{1^2} - \frac{1}{2^2}\right)$$

Sommerfeld:

$$E = \frac{\mu e^4}{2\hbar n^2}, \quad L_z = m\hbar, \quad |\vec{L}| = l\hbar$$

$$\lambda = \frac{h}{p}, \quad \nu = \frac{E}{h}, \quad \Delta E \Delta t \geq \frac{\hbar}{2}, \quad \Delta x \Delta p \geq \frac{\hbar}{2}$$

Ecuación de Schödinger

$$i\hbar \frac{\partial \psi(x,t)}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \psi(x,t)}{\partial x^2} + V(x)\psi(x,t)$$

$$-\frac{\hbar^2}{2\mu} \frac{d^2 X(x)}{dx^2} + V(x)X(x) = EX(x), \quad \psi(x,t) = X(x)e^{-iE/\hbar t}$$

$$p \rightarrow -i\hbar \frac{\partial}{\partial x}, \quad E = \frac{p^2}{2\mu} + V \rightarrow -\frac{\hbar^2}{2\mu} \frac{\partial^2}{\partial x^2} + V$$

$$E \rightarrow i\hbar \frac{\partial}{\partial t}, \quad \langle E \rangle = \int \psi^*(x,t) E \psi(x,t) dx$$

$$\langle E^l \rangle = \sum |a_n|^2 E_n^l, \quad \Delta E^2 = \langle E^2 \rangle - \langle E \rangle^2$$

Pozo infinito (0, a) :

$$E_n = \frac{\hbar^2 n^2}{8\mu a^2}, \quad \varphi_n(x) = \sqrt{2/a} \sin \frac{n\pi x}{a}$$

$(-l, l) :$

$$E_n = \frac{\hbar^2 n^2 \pi^2}{8\mu l^2}, \quad \varphi_n(x) = \sqrt{1/l} \sin \frac{n\pi}{2l}(x+l)$$

Pozo $V_0(-a/2, a/2) : k^2 = \frac{2\mu}{\hbar^2} E, \quad \alpha^2 = \frac{2\mu}{\hbar^2}(V_0 - E)$
Pares

$$\operatorname{tg} \frac{ka}{2} = \frac{\alpha}{k}, \quad A = \frac{\sqrt{2/a}}{\sqrt{1 + \frac{\sin ka}{ka} + \frac{2}{\alpha a} \cos^2 \frac{ka}{2}}}$$

$$X(|x|) = \begin{cases} A \cos k|x| & \text{si } |x| \leq a/2 \\ A \cos \frac{ka}{2} e^{-\alpha|x-a/2|} & \text{si } |x| \geq a/2 \end{cases}$$

Impares

$$\operatorname{tg} \frac{ka}{2} = -\frac{k}{\alpha}, \quad C = \frac{\sqrt{2/a}}{\sqrt{1 - \frac{\sin ka}{ka} + \frac{2}{\alpha a} \sin^2 \frac{ka}{2}}}$$

$$X(x) = \begin{cases} C \sin k|x| & \text{si } |x| \leq a/2 \\ C \sin \frac{ka}{2} e^{-\alpha(x-a/2)} & \text{si } |x| \geq a/2 \\ -X(-x) & \text{si } x < 0 \end{cases}$$

$$\theta = \frac{ka}{2}, \quad \theta_0 = \sqrt{\frac{\mu V_0 a^2}{2\hbar^2}}, \quad V_0 = \frac{2\hbar^2}{\mu a^2} \theta_0^2, \quad E = V_0 \left(\frac{\theta}{\theta_0} \right)^2$$

$$\text{p) } \operatorname{tg} \theta = \sqrt{\left(\frac{\theta_0}{\theta} \right)^2 - 1},$$

$$\text{imp) } \operatorname{cotg} \theta = -\sqrt{\left(\frac{\theta_0}{\theta} \right)^2 - 1}$$

Oscilador armónico $V(x) = \frac{1}{2} k x^2 = \frac{1}{2} \mu \omega^2 x^2$

$$\alpha = \sqrt{\frac{2\mu\omega}{\hbar}}, \quad E_n = \left(n + \frac{1}{2} \right) \hbar\omega$$

$$X_n(x) = \left(\frac{\sqrt{2\pi}}{\alpha} 2^n n! \right)^{-1/2} e^{-\frac{1}{4}\xi^2} H_n \left(\frac{\xi}{\sqrt{2}} \right),$$

$$\xi = \alpha x$$

Hermite:

$$H_n(z) = (-1)^n e^{z^2} \frac{d^n}{dz^n} e^{-z^2}$$

Dispersión en un pozo

$$k'^2 = \frac{2\mu}{\hbar^2}(V_0 - E), \quad k^2 = \frac{2\mu}{\hbar^2} E$$

$$T = \left| \frac{F}{A} \right|^2 = \frac{4k^2 k'^2}{(k^2 - k'^2) \sin^2 ka + 4k^2 k'^2}$$

$$R = \left| \frac{B}{A} \right|^2 = 1 - T = \frac{(k^2 - k'^2) \sin^2 ka}{(k^2 - k'^2) \sin^2 ka + 4k^2 k'^2}$$

Barrera de potencial $V_0(-a/2, a/2) :$

$$\beta^2 = -k'^2 = \frac{2\mu}{\hbar}(V_0 - E), \quad k^2 = \frac{2\mu}{\hbar} E$$

$$(E \leq V_0) T = \frac{4k^2 \beta^2}{(k^2 + \beta^2)^2 \sinh^2 \beta a + 4\beta^2 k^2}, \quad R = 1 - T$$

$$(E \geq V_0) T = \frac{4k^2 k'^2}{(k^2 - k'^2)^2 \sin^2 k'a + 4k'^2 k^2}, \quad R = 1 - T$$

$$\left[-\frac{\hbar^2}{2\mu} \nabla^2 + V(\vec{r}) \right] X(\vec{r}) = EX(\vec{r}), \quad \vec{p} \rightarrow -i\hbar \vec{\nabla}$$

$$L_x = i\hbar \left(\sin \varphi \frac{\partial}{\partial \theta} - \operatorname{cotg} \theta \cos \varphi \frac{\partial}{\partial \varphi} \right)$$

$$L_y = i\hbar \left(-\cos \varphi \frac{\partial}{\partial \theta} + \operatorname{cotg} \theta \sin \varphi \frac{\partial}{\partial \varphi} \right)$$

$$L_z = -i\hbar \frac{\partial}{\partial \varphi}, \quad \nabla^2 = \frac{1}{r^2} \frac{\partial^2}{\partial r^2} r - \frac{1}{r^2} \frac{L^2}{\hbar^2}$$

$$L^2 = -\hbar \left[\frac{1}{\sin^2 \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial}{\partial \theta} \right) + \frac{1}{\sin^2 \theta} \frac{\partial^2}{\partial \theta^2} \right]$$

$$L_{\pm} = L_x \pm iL_y, \quad L_{\pm} Y_{l,m} = \sqrt{l(l+1) - m(m \pm 1)} \hbar Y_{l,m \pm 1}$$

Armónicos esféricos y Legendre

$$Y_{lm}(\theta, \varphi) = \sqrt{\frac{2l+1}{4\pi} \frac{(l-m)!}{(l+m)!}} P_l^m(\cos \theta) e^{im\varphi}$$

$$P_l^m(z) = \frac{(-1)^m}{2^l l!} (1-z^2)^{m/2} \frac{d^{l+m}}{dz^{l+m}} (z^2 - 1)^l$$

Cuantización

$$L^2 Y = l(l+1) \hbar^2 Y, \quad L_z Y = m \hbar Y, \quad |m| \leq l$$

$$\left[-\frac{\hbar^2}{2\mu} \frac{d^2}{dr^2} + V(r) + \frac{l(l+1)\hbar^2}{2\mu r^2} \right] R_{nl}(r) = E_n R_{nl}(r)$$

Átomos con un electrón ($l = 0, 1, \dots, n - 1$)

$$X(x) = \frac{R(r)}{r} Y(\theta, \varphi), \quad a = \frac{\hbar}{\mu Z e^2}, \quad E = -\frac{\mu Z^2 e^4}{2\hbar^2} \frac{1}{n^2}$$

$$\left[\frac{d^2}{d\xi^2} - \frac{l(l+1)}{\xi^2} + \frac{2}{\xi} - \frac{1}{\lambda^2} \right] R(r) = 0, \quad \xi = r/a$$

$$R_{nl}(r) = \sqrt{\frac{(n-l-1)!}{2na[(n+l)!]^3}} \left(\frac{2}{n}\right)^{2l+3} \xi^{l+1} \mathcal{L}_{n+l}^{2l+1} \left(\frac{2\xi}{n}\right) e^{-\xi/n}$$

Laguerre

$$\mathcal{L}_p^q(z) = \frac{d^q}{dz^q} \mathcal{L}_p(z) \quad \mathcal{L}_p(x) = e^z \frac{d^p}{dz^p} (z^p e^{-z})$$

$$h = 6,626 \times 10^{-34} \text{Js} = 4,136 \times 10^{-15} \text{eVs}$$

$$\hbar = 1,055 \times 10^{-34} \text{Js} = 6,582 \times 10^{-16} \text{eVs}$$

$$|E_1| = \frac{1}{[4\pi\epsilon_0]} \frac{e^2}{2a} = 13,61 \text{eV}$$

$$a_0 = [4\pi\epsilon_0] \frac{\hbar^2}{m_e e^2} = 0,529 \text{Å}$$

$$\lambda_C = \frac{h}{m_e c} = 0,0243 \text{Å}$$

$$m_e c^2 = 0,511 \text{MeV}, \quad m_p = 1836 m_e$$