Ampliación de electromagnetismo

pod

Otoño, 2001

1 Propagación del campo

Maxwell

$$\vec{\nabla}\cdot\vec{D}=\rho\ ,\ \vec{\nabla}\times\vec{H}=\vec{\jmath}+\epsilon_0+\frac{\partial\vec{D}}{\partial t}$$

$$\vec{\nabla}\cdot\vec{B}=0\ ,\ \vec{\nabla}\times\vec{E}=-\frac{\partial\vec{B}}{\partial t}$$

$$\hat{n}\cdot(\vec{D}_+-\vec{D}_-)=\sigma\ ,\ \hat{n}\cdot(\vec{B}_+-\vec{B}_-)=0$$

$$\hat{n}\times(\vec{E}_+-\vec{E}_-)=0\ ,\ \hat{n}\times(\vec{H}_+-\vec{H}_-)=\vec{\kappa}$$

$$\vec{D}=\epsilon_0\vec{E}+\vec{P}=\epsilon\ ,\ \vec{B}=\mu_0(\vec{H}+\vec{M})=\mu\vec{H}$$

Poyting $\vec{N}=\vec{E}\times\vec{H}$, conservación

$$\int_v\vec{E}'\cdot\vec{\jmath}\mathrm{d}v=\int_v\frac{j^2}{\gamma}\mathrm{d}v+\frac{\partial}{\partial t}\int_v\frac{1}{2}(\vec{E}\vec{D}+\vec{B}\vec{H})\mathrm{d}v+\oint_S\vec{N}\mathrm{d}\vec{S}$$

Propagación

$$\Delta\vec{E}-\mu\gamma\frac{\partial\vec{E}}{\partial t}-\mu\epsilon\frac{\partial^2\vec{E}}{\partial t^2}=0\ ,$$

$$\Delta\vec{H}-\mu\gamma\frac{\partial\vec{H}}{\partial t}-\mu\epsilon\frac{\partial^2\vec{H}}{\partial t^2}=0$$

$$\Delta\vec{E}+\omega^2\epsilon\mu\left(1-j\frac{\gamma}{\omega\epsilon}\right)\vec{E}=0$$

Onda progresiva

$$E_i=E_o\mathrm{e}^{j(\omega t-kx)}\ ,\ \vec{B}=\frac{k}{\omega}\hat{n}\times\vec{E}\ ,\ k=\omega\sqrt{\epsilon\mu}$$

Conductores $K^2=\epsilon\mu\omega^2-j\omega\mu\gamma=\tilde{\epsilon}\mu\omega^2$

$$\beta=k\sqrt{\frac{1}{2}\left[\sqrt{1+\left(\frac{\gamma}{\epsilon\mu}\right)^2}+1\right]}\ ,$$

$$K=\beta-j\alpha\ ,\ \mathrm{tg}\delta=\frac{\gamma}{\epsilon\mu}$$

$$\alpha=k\sqrt{\frac{1}{2}\left[\sqrt{1+\left(\frac{\gamma}{\epsilon\mu}\right)^2}-1\right]}\ ,$$

$$\lambda=\frac{2\pi}{\beta}\ ,\ K=M\mathrm{e}^{j\phi}$$

$$K=_{(\gamma\gg)}\frac{1}{\sqrt{2}}(1-j)\sqrt{\omega\mu\gamma}$$

Impedancia intrínseca

$$\eta=\frac{E_x}{H_y}=\frac{\omega\mu}{K}=_{(\gamma=0)}\sqrt{\frac{\mu}{\epsilon}}=\sqrt{\frac{\omega\mu}{2\gamma}}(1+j)$$

2 Líneas de transmisión

Ecuaciones generales

$$\frac{\partial V}{\partial x}=Ri+L\frac{\partial i}{\partial t}\ ,\ \frac{\partial i}{\partial x}=GV+C\frac{\partial V}{\partial t}$$

Sin pérdidas ($R=G=0$)

$$\frac{\partial^2 V}{\partial x^2}=LC\frac{\partial^2 V}{\partial t^2}\ ,\ \frac{\partial^2 i}{\partial x^2}=LC\frac{\partial^2 i}{\partial t^2}\ ,\ v=\frac{1}{\sqrt{LC}}=\frac{1}{\sqrt{\epsilon\mu}}$$

Sinusoidal

$$\frac{\mathrm{d}V}{\mathrm{d}x}=(R+j\omega L)I=zI\ ,\ \frac{\mathrm{d}I}{\mathrm{d}x}=(G+j\omega C)V=yV$$

Cte. propagación y impedancia característica

$$\gamma^2=yz\ ,\ \gamma=\alpha+j\beta\ ,\ \lambda=\frac{2\pi}{\beta}\ ,\quad \frac{1}{Z_0}=\frac{\gamma}{z}=\sqrt{\frac{y}{z}}$$

$$V(x)=V_R\left\{\cosh\gamma x+\frac{Z_0}{Z_r}\sinh\gamma x\right\}$$

$$I(x) = I_r \left\{ \cosh \gamma x + \frac{Z_0}{Z_r} \sinh \gamma x \right\}$$

Impedancia de línea

$$Z(x) = \frac{V(x)}{I(x)} = Z_0 \frac{Z_R + Z_0 \mathrm{tgh} \gamma x}{Z_0 + Z_R \mathrm{tgh} \gamma x}$$

Pérdidas (1r orden)

$$\alpha = \frac{R}{2Z_0} + \frac{1}{2} G Z_0 \ , \ \beta = \omega \sqrt{LC} \ , Z_0 = \sqrt{L/C}$$

Vientres i nodos de tensión (intensidad, intercambiados)

$$\phi_0 - \beta x_M = 2n\pi \ , \phi_0 - 2\beta x_m = (2n+1)\pi \ ,$$

$$|x_M - x_m| = \lambda/4$$

Coef. reflexión

$$r_R = \frac{Z_R - Z_0}{Z_R + Z_0} = |r_R| \mathrm{e}^{i\phi_0} \ ,$$

$$Z(x) = Z_0 \frac{1 + r_R \mathrm{e}^{-2\gamma x}}{1 - r_R \mathrm{e}^{-2\gamma x}}$$

$$V(x) = V_R' \mathrm{e}^{\gamma x} \left\{ 1 + |r_R| \mathrm{e}^{j\phi_0} \mathrm{e}^{-2\gamma x} \right\}$$

$$I(x) = \frac{V_r'}{Z_0} \mathrm{e}^{\gamma x} \left\{ 1 - |r_R| \mathrm{e}^{j\phi_0} \mathrm{e}^{-2\gamma x} \right\}$$

Relación ondas estacionarias

$$\rho = S = \frac{V_M}{V_m} = \frac{1 + |r_R|}{1 - |r_R|} \ , \ \bar{z}_M = \rho = \frac{1}{\bar{z}_m}$$

Adaptación impedancias $Z(l) = Z_g^*$

Cuarto de onda

$$Z(\lambda/4) = Z_0^2 / Z_R = Z_g^* \ , \ \bar{z}_{\mathrm{ent}} = \frac{1}{\bar{z}_R}$$

Media onda

$$Z(\lambda/2) = Z_R$$

Stub $\bar{Y}(l_1) = 1 + \sigma_l$

$$\bar{y} = 1/\bar{z} = \rho + j\sigma \ , \ Y = Y_{\text{línea}}(l_1) + j(\sigma_{\text{línea}} + \sigma_{\text{stub}})$$

$$l_1 = \frac{\lambda}{2\pi} \left\{ \phi_0 - \arccos(-|r_R|) \right\}$$

$$\text{c.c. } \mathrm{cotg} \frac{2\pi}{\lambda} l_2 = \frac{-2|r_R|}{\sqrt{1 - |r_R|^2}} \ ,$$

$$\text{c.o. } \mathrm{tg} \frac{2\pi}{\lambda} l_2 = \frac{2|r_R|}{\sqrt{1 - |r_R|^2}}$$

Smith

$$\psi = \phi_0 - 2\beta x \ , \ M \mathrm{e}^{j\psi} = p + jq \ , \ \bar{z} = \frac{1 + p + jq}{1 - (p + jq)} = r + jx$$

$$r = \frac{(1+p)(1-p) - q^2}{(1-p)^2 + q^2} \ , \ x = \frac{2q}{(1-p)^2 + q^2}$$

Atenuación $1\mathrm{dB} = 1\mathrm{Np}/8,686$

$$\alpha \Delta z = 10 \log \frac{P_0}{P} = 20 \log \frac{E_0}{E} \ ,$$

$$E = E_0 \mathrm{e}^{-\alpha(z - z_0)} \ , \ P = P_0 \mathrm{e}^{-2\alpha(z - z_0)}$$

3 Guias conductores

Condición de contorno $\frac{\partial H_\tau}{\partial n} = 0$

Impedancia

$$Z(z) = Z_{g1} \frac{Z_{g2} - Z_{g1} \mathrm{tgh}(j k_g z)}{Z_{g1} - Z_{g2} \mathrm{tgh}(j k_g z)} = Z_{g1} \frac{1 + \Gamma \mathrm{e}^{2jk_{g1}z}}{1 - \Gamma \mathrm{e}^{j2k_{g1}z}}$$

Coef. reflexión

$$\Gamma = \frac{E_r}{E_i} = \frac{Z_{g2} - Z_{g1}}{Z_{g2} + Z_{g1}}$$

Modos TE

$$E_z = 0 \ , \ E_s = -j \frac{\omega\mu}{k_s^2} \vec{\nabla}_s \times \vec{H}_z = -\frac{\omega\mu}{k_s^2} \vec{H}_s \times \vec{a}_z$$

$$\vec{H}_s = -j \frac{k_g}{k_s} \vec{\nabla}_s H_z$$

Modos TM

$$H_z = 0 \ , v E_s = -j \frac{k_g}{k_s} \vec{\nabla}_s E_z \ , \ \vec{H}_s = \frac{\omega\epsilon}{k_s} \vec{E}_s \times \vec{a}_z$$

Guia rectangular:

$$k_s^2 = \pi^2 \left[\left(\frac{m}{a} \right)^2 + \left(\frac{n}{b} \right)^2 \right] \ , \ \lambda_c = \frac{2\pi}{k_s} = \frac{2}{\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}}$$

$$k_x = m\pi/a \ , \ k_y = m\pi/b \ , \ \lambda_{\max} = 2 \max(a, b)$$

G. rectangular, modos TE_{mn}

$$
\begin{aligned}
H_x &= j\left(\frac{\lambda_c}{2\pi}\right)^2 k_x H_0 \sin k_x x \cos k_y y \ \mathrm{e}^{-j(k_g z-\omega t)} \\
H_y &= -j\left(\frac{\lambda_c}{2\pi}\right)^2 k_y H_0 \cos k_x x \sin k_y y \ \mathrm{e}^{-j(k_g z-\omega t)} \\
H_z &= H_0 \cos k_x x \cos k_y y \mathrm{e}^{-j(k_g z-\omega t)} \\
E_x &= j\omega\mu H_0 \left(\frac{\lambda_c}{2\pi}\right)^2 k_y \cos k_x x \cos k_y y \ \mathrm{e}^{-j(k_g z-\omega t)} \\
E_y &= -j\omega\mu H_0 \left(\frac{\lambda_c}{2\pi}\right)^2 k_x \sin k_x x \cos k_y y \ \mathrm{e}^{-j(k_g z-\omega t)} \\
E_z &= 0
\end{aligned}
$$

G. rectangular, modos TM_{mn} ($\nexists\ TM_{m0}, TM_{0n}$)

$$
\begin{aligned}
E_x &= -jk_g k_x \left(\frac{\lambda_c}{2\pi}\right)^2 E_0 \cos k_x x \sin k_y x \ \mathrm{e}^{-j(k_g z-\omega t)} \\
E_y &= -jk_g k_y \left(\frac{\lambda_c}{2\pi}\right)^2 E_0 \sin k_x x \cos k_y y \ \mathrm{e}^{-j(k_g z-\omega t)} \\
E_z &= E_0 \sin k_x \sin k_y y \ \mathrm{e}^{-j(k_g z-\omega t)} \\
H_x &= j\omega\epsilon k_y \left(\frac{\lambda_c}{2\pi}\right)^2 E_0 \sin k_x x \cos k_y y \ \mathrm{e}^{-j(k_g z-\omega t)} \\
H_y &= -j\omega\epsilon k_x \left(\frac{\lambda_c}{2\pi}\right)^2 E_0 \cos k_x x \sin k_y y \ \mathrm{e}^{-j(k_g z-\omega t)} \\
H_z &= 0
\end{aligned}
$$

Guias planes: $TEM = TM_0$ ($f_c = 0$)

$$
k_y = \frac{n\pi}{b}\ ,\ k_g = \sqrt{\omega^2\epsilon\mu - \left(\frac{n\pi}{b}\right)^2}\ ,\ \lambda_c = \frac{2b}{n}
$$

G. plana, modos TM_n

$$
\begin{aligned}
E_x &= 0 \\
E_y &= -jk_g \frac{\lambda_c}{2\pi} E_0 \cos k_y y \ \mathrm{e}^{-j(k_g z-\omega t)} \\
E_z &= E_0 \sin k_y y \ \mathrm{e}^{-j(k_g z-\omega t)} \\
H_x &= -jk_g \frac{\lambda_c}{2\pi} E_0 \cos k_y y \ \mathrm{e}^{-j(k_g z-\omega t)} \\
H_y &= 0 \\
H_z &= 0
\end{aligned}
$$

G. plana, modos TE_n ($\nexists TE_0$)

$$
\begin{aligned}
E_x &= j\omega\mu \frac{\lambda_c}{2\pi} H_0 \sin k_y y \ \mathrm{e}^{-j(k_g z-\omega t)} \\
E_y &= 0 \\
E_z &= 0 \\
H_x &= 0 \\
H_y &= -jk_g \frac{\lambda_c}{2\pi} H_0 \sin k_y y \ \mathrm{e}^{-j(k_g z-\omega t)} \\
H_z &= H_0 \cos k_y y \mathrm{e}^{-j(k_g z-\omega t)}
\end{aligned}
$$

Guia cilíndrica circular: Modos TE_{ln}

$$
k_s\Big|_{TE_{ln}} = \frac{Z^l_{\mathrm{ero}}\mathcal{J}'_n}{r_0}\ ,\ Z^1_{\mathrm{ero}}J'_1 = 1,84\ ,\ \lambda_{\min} = 3,41r_0
$$

$$
\begin{aligned}
E_r &= j\frac{\omega\mu}{rk_s^2} n H_0 \mathcal{J}_n(k_s r) \sin n\varphi \ \mathrm{e}^{-j(k_g z-\omega t)} \\
E_\varphi &= j\frac{\omega\mu}{k_s} H_0 \mathcal{J}'_n(k_s r) \cos n\varphi \ \mathrm{e}^{-j(k_g z-\omega t)} \\
E_z &= 0 \\
H_r &= -j\frac{k_g}{k_s} H_0 \mathcal{J}'_n(k_s r) \cos n\varphi \ \mathrm{e}^{-j(k_g z-\omega t)} \\
H_\varphi &= j\frac{k_g}{rk_s^2} n H_0 \mathcal{J}_n(k_s r) \sin n\varphi \ \mathrm{e}^{-j(k_g z-\omega t)} \\
H_z &= H_0 \mathcal{J}_n(k_s r) \cos n\varphi \ \mathrm{e}^{-j(k_g z-\omega t)}
\end{aligned}
$$

Guia cilíndrica circular. Modos TM_{ln}

$$
k_s\Big|_{TM_{ln}} = \frac{Z^l_{\mathrm{ero}}\mathcal{J}_n}{r_0}\ ,\ \lambda_c = \frac{2\pi}{k_s}
$$

$$
\begin{aligned}
E_r &= -j\frac{k_g}{k_s} E_0 \mathcal{J}'_n(k_s r) \cos n\varphi \ \mathrm{e}^{-j(k_g z-\omega t)} \\
E_\varphi &= j\frac{k_g}{rk_s^2} n E_0 \mathcal{J}_n(k_s r) \sin n\varphi \ \mathrm{e}^{-j(k_g z-\omega t)} \\
E_z &= E_0 \mathcal{J}_n(k_s r) \cos n\varphi \mathrm{e}^{-j(k_g z-\omega t)} \\
H_r &= -j\frac{\omega\epsilon}{rk_s^2} n E_0 \mathcal{J}_n(k_s r) \sin n\varphi \ \mathrm{e}^{-j(k_g z-\omega t)} \\
H_\varphi &= j\frac{\omega\epsilon}{k_s} E_0 \mathcal{J}'_n(k_s r) \cos n\varphi \ \mathrm{e}^{-j(k_g z-\omega t)} \\
H_z &= 0
\end{aligned}
$$

Guia coaxial: ($a < b$, $ak_s, bk_s \ll 1$) Modo TE_l i TM_l

$$
k_s \sim \frac{l\pi}{b-a}\ ,\ \lambda_c = 2l(b-a)
$$

Modo TEM ($\lambda_c \to \infty$)

$$
k_g = \omega\sqrt{\epsilon\mu}\ ,\ v = 1/\sqrt{\epsilon\mu}\ ,\ Z_h = \eta = \sqrt{\mu/\epsilon}
$$

4 Cavidades resonantes

$$\frac{\mathrm{d}H_\tau}{\mathrm{d}n} = 0 \ , \ \vec{E} \| S \ , \ \vec{H} \perp S$$

Factor calidad

$$Q = \omega \frac{W}{P_{\mathrm{perd}}} \ , \ W = \frac{1}{2} \int_v (\epsilon E^2 + \mu H^2) \mathrm{d}v$$

Cavidad rectangular: ($\nexists TE_{000}$)

$$k_x = \frac{l\pi}{a} \ , \ k_y = \frac{m\pi}{b} \ , \ k_z = \frac{n\pi}{c} \ ,$$

$$\omega_{lmn} = v k_{lmn} = 2\pi f_{lmn}$$

$$k_{lmn} = \pi \sqrt{\left(\frac{l}{a}\right)^2 + \left(\frac{m}{b}\right)^2 + \left(\frac{n}{c}\right)^2}$$

$$Q = \frac{\omega \mu V_{\mathrm{olum}}}{2 R_s S_{\mathrm{total}}} \ , \ R_s = \frac{1}{\delta\gamma} = \sqrt{\frac{\omega\mu}{2\gamma}}$$

$$\begin{aligned}
E_x &= E_x \cos k_x x \sin k_y y \sin k_z z \ \mathrm{e}^{j\omega t} \\
E_y &= E_y \sin k_x x \cos k_y y \sin k_z z \ \mathrm{e}^{j\omega t} \\
E_z &= E_z \sin k_x x \sin k_y y \cos k_z z \ \mathrm{e}^{j\omega t} \\
H_x &= \frac{k_3 E_2 - k_2 E_3}{j\omega\mu} \sin k_x \cos k_y y \cos k_z z \ \mathrm{e}^{j\omega t} \\
H_y &= \frac{k_1 E_3 - k_1 E_3}{j\omega\mu} \cos k_x \sin k_y y \cos k_z z \ \mathrm{e}^{j\omega t} \\
H_z &= \frac{k_2 E_1 - k_1 E_2}{j\omega\mu} \cos k_x \cos k_y y \sin k_z z \ \mathrm{e}^{j\omega t}
\end{aligned}$$

Cavidad cilíndrica circular:
Modos TM_{mln}

$$k_s = \frac{Z^l_{\mathrm{ero}} \mathcal{J}_m}{r_0} \ , \ k_z = \frac{n\pi}{c} \ , \ \omega_{mln} = v\sqrt{k_z^2 + k_s^2}$$

$$Q = \frac{\pi \eta r_0 c}{\lambda R_s (c + r_0)} \ , \ R_s = \frac{1}{\delta\gamma} = \frac{\alpha}{\gamma}$$

$$\begin{aligned}
E_r &= -k_s k_z \mathcal{J}'_m(k_s r) \cos m\varphi \cos k_z z \ \mathrm{e}^{j\omega t} \\
E_\varphi &= \frac{k_z m}{r} \mathcal{J}_m(k_s r) \sin m\varphi \sin k_z z \ \mathrm{e}^{j\omega t} \\
E_z &= k_s^2 \mathcal{J}_m(k_s r) \cos m\varphi \cos k_z z \ \mathrm{e}^{j\omega t} \\
H_r &= -j \frac{\omega \epsilon m}{r} \mathcal{J}_m(k_s r) \sin m\varphi \cos k_z z \ \mathrm{e}^{j\omega t} \\
H_\varphi &= -j\omega\epsilon k_s \mathcal{J}'_m(k_s r) \cos m\varphi \cos k_z z \ \mathrm{e}^{j\omega t} \\
H_z &= 0
\end{aligned}$$

Modos TE_{mln} ($\nexists TM_{0l0}$)

$$k_s = \frac{Z^l_{\mathrm{ero}} \mathcal{J}'_m}{r_0} \ , \ k_z = \frac{n\pi}{c} \ , \ \omega_{mln} = v\sqrt{k_z^2 + k_s^2}$$

$$\begin{aligned}
E_r &= j \frac{\omega \mu m}{r} \mathcal{J}_m(k_s r) \sin m\varphi \sin k_z z \ \mathrm{e}^{j\omega t} \\
E_\varphi &= j\omega\mu k_s \mathcal{J}'_m(k_s r) \cos m\varphi \sin k_z z \ \mathrm{e}^{j\omega t} \\
E_z &= 0 \\
H_r &= k_z k_s \mathcal{J}'_m(k_s r) \cos m\varphi \cos k_z z \ \mathrm{e}^{j\omega t} \\
H_\varphi &= -\frac{k_z m}{r} \mathcal{J}_m(k_s r) \sin m\varphi \cos k_z z \ \mathrm{e}^{j\omega t} \\
H_z &= k_s^2 \mathcal{J}_m(k_s r) \cos m\varphi \sin k_z z \ \mathrm{e}^{j\omega t}
\end{aligned}$$

5 Guias dieléctricas

Guia plana: Modos TE

$$\alpha_c = \omega^2(\mu_1\epsilon_1 - \mu_2\epsilon_2) \ , \ \alpha_c < \frac{\pi}{a}$$

Modos TE, par

$$E_y\Big|_1 = E_1 \cos \alpha x \mathrm{e}^{j(\omega t - k_z z)} \ ,$$

$$E_y\Big|_2 = E_2 \mathrm{e}^{-\beta x} \mathrm{e}^{j(\omega t - k_z z)}$$

$$H_x\Big|_1 = -\frac{k_z}{\omega\mu} E_y \ ,$$

$$H_z\Big|_1 = -j\frac{\alpha}{\omega\mu} E_1 \sin \alpha x \ \mathrm{e}^{j(\omega t - k_z z)}$$

$$H_x\Big|_1 = -\frac{k_z}{\omega\mu} E_y \ ,$$

$$H_z\Big|_2 = -j\frac{\beta}{\mu\omega} E_y$$

condiciones de propagación

$$\epsilon_1 > \epsilon_2 \ , \mathrm{tg}\alpha\frac{a}{2} = \frac{\beta}{\alpha} = \sqrt{\frac{\omega(\mu_1\epsilon_1 - \mu_2\epsilon_2)}{\alpha^2} - 1}$$

Modos TE, impar

$$E_y\Big|_1 = E_1 \sin \alpha x \mathrm{e}^{j(\omega t - k_z z)} \ ,$$

$$H_z\Big|_1 - j\frac{\alpha}{\omega\mu} E_1 \sin \alpha x \ \mathrm{e}^{j(\omega t - k_z z)}$$

condiciones de propagación

$$\epsilon_1 > \epsilon_2 \ , -\text{cotg}\alpha\frac{a}{2} = \sqrt{\frac{\omega^2(\mu_1\epsilon_1 - \mu_2\epsilon_2)}{\alpha^2} - 1}$$

Modos TM, par

$$H_y\Big|_1 = H_1 \cos\alpha x \mathrm{e}^{j(\omega t - k_z z)} \ ,$$

$$H_y\Big|_2 = H_2 \mathrm{e}^{-\beta x}\mathrm{e}^{j(\omega t - k_z z)}$$

$$\vec{\nabla}\times\vec{H} = \frac{\partial\vec{E}}{\partial t} \ , \ \text{tg}\alpha\frac{a}{2} = \frac{\epsilon_1}{\epsilon_2}\frac{\beta}{\alpha}$$

Impar

$$H_y\Big|_1 = H_1 \sin\alpha x \mathrm{e}^{j(\omega t - k_z z)} \ , \ -\text{cotg}\alpha\frac{a}{2} = \frac{\epsilon_1}{\epsilon_2}\frac{\beta}{\alpha}$$

Fibras ópticas: Modo TM

$$E_z\Big|_1 = E_1\mathcal{J}_n(\alpha r)\cos n\varphi \ \mathrm{e}^{j(\omega t - k_z z)} \ ,$$

$$\vec{E}_s = -j\frac{k_g}{k_s^2}\vec{\nabla}_s\vec{E}_z$$

$$E_z\Big|_3 = E_2 K_n(\beta r)\cos n\varphi E_\varphi\Big|_1 \ ,$$

$$\vec{H} = \frac{\omega\epsilon}{k_g}\vec{a}_z \times \vec{E}_s$$

$$\frac{\epsilon_1}{\alpha}\frac{\mathcal{J}_{n-1}(\alpha a) - \mathcal{J}_{n+1}(\alpha a)}{\mathcal{J}_n(\alpha a)} = -\frac{\epsilon_2}{\beta}\frac{\mathcal{K}_{n-1}(\beta a) + \mathcal{K}_{n+1}(\beta a)}{\mathcal{K}_n(\beta a)}$$

$$\alpha^2 + \beta^2 = \omega^2(\epsilon_1\mu_1 - \epsilon_2\mu_2)$$

Corte $\lambda_c = 2\pi/\alpha_c$, un modo $\alpha_c > \pi/a$
Multimodo ($a \gg \lambda$, $\epsilon_1 \gg \epsilon_2$)

$$\frac{\epsilon_1}{\alpha}\text{tg}\left(\alpha a - \frac{n\pi}{2} - \frac{\pi}{2}\right) = -\frac{\epsilon_2}{\beta}$$

Óptica integrada:

$$v_{ph} = \frac{\omega}{k} = \frac{1}{\sqrt{\epsilon_1\mu_1}}\left[1 - \frac{\alpha^2}{\omega^2\mu_1\epsilon_1}\right]^{-1/2} \approx v_1\left[1 + \frac{k'^2}{8\pi^2}\frac{\lambda^2}{a^2}\right]$$

$$n_{ef} = \frac{c}{v_1}\frac{v_1}{v_{ph}} \approx \frac{n_1}{1 + \frac{\alpha^2}{\omega^2\mu_1\epsilon_1}}$$

6 Movimiento de cargas

Óptica electrostática:

$$\sqrt{V_1}\sin\alpha_1 = \sqrt{V_2}\sin\alpha_2 \ , \ \Delta V = 0 \ , \ E_r = \frac{r}{2}V''$$

Ec. básica

$$\frac{\mathrm{d}^2 r}{\mathrm{d}z^2} + \frac{V'}{2V}\frac{\mathrm{d}r}{\mathrm{d}z} + \frac{r}{4}\frac{V''}{V} = 0 \ ,$$

$$r(z) = C_1 r_1(z) + C_2 r_2(r)$$

Ec. Helmholtz-Lagrange

$$h_a\gamma_a\sqrt{V(a)} = h_b\gamma_b\sqrt{V(b)} \ ,$$

$$r_1'(b) = \Gamma' \ , \ r_2(b) = \beta'$$

Lente delgada:

$$\frac{1}{a} + \frac{1}{b} = \frac{1}{f} = \frac{1}{4\sqrt{V_0}}\int_{a'}^{b'}\frac{V''}{V}\mathrm{d}z \ ,$$

$$\frac{1}{f_i} = \frac{1}{4\sqrt{V_i}}\int_{a'}^{b'}\frac{V''}{V}\mathrm{d}z$$

$$\frac{f_a}{a} + \frac{f_b}{b} = 1 \ , \ \frac{f_a}{f_b} = \sqrt{\frac{V_a}{V_b}}$$

Campos magnéticos

$$\rho = \frac{mv_\perp}{qB} \ , \ T = \frac{2\pi m}{qB} \ , \ l = \frac{2\pi m}{qB}v_0\cos\alpha$$

$$\Delta = BC = 4\rho\sin^2\frac{\psi}{2} \approx \rho\psi^2$$

Campo axial

$$B_r = -\frac{r}{2}\frac{\partial B_z}{\partial z} \ ,$$

$$\Delta\varphi = \frac{e}{2mv}\int_a^b B\mathrm{d}z \ , \ \frac{\mathrm{d}^2 r}{\mathrm{d}z^2} + \frac{eB^2}{mV}r = 0$$

Lente delgada

$$\frac{1}{a} + \frac{1}{b} = \frac{e}{8mV}\int_a^b B^2\mathrm{d}z = \frac{1}{f} > 0$$

Lentamente variables (invar. adiabático)

$$\rho\frac{|\nabla\vec{B}|}{B} \ll 1 \ , \ m_b = \frac{W_\perp}{B} = \frac{1}{2}mv_\perp^2 = cte \ , \ \frac{\sin^2\alpha}{B} = cte$$

Botella magnética

$$\sqrt{\frac{B_0}{B_m}} < \sin\alpha_0$$

Campo transversal: Condensador plano ($\vec{E} = -E\vec{a}_y$)

$$y = x\mathrm{tg}\beta - \frac{qE}{2m}\frac{x^2}{(v_0\cos\beta)^2} \ ,$$

$$(x,y)_{\mathrm{foc}} = \frac{v_0^2}{a}\left(\mathrm{cotg}\beta,\ 1 - \frac{1}{\cos^2\beta}\right)$$

Condensador cilíndrico ($r = r_0 + u$):

$$E = \alpha/r = \frac{\Delta V}{r\ln(r_2/r_1)} \ , \ \ddot{u} + \frac{2q\alpha}{mr_0^2}u = 0 \ ,$$

$$u = \frac{\epsilon r_0}{\sqrt{2}}\sin\sqrt{2}\varphi$$

$$\varphi_{\mathrm{foc}} = \frac{\pi}{\sqrt{2}} \ , \ \varphi_{\mathrm{par}} = \frac{\pi}{2\sqrt{2}}$$

lente cuadrupolar eléctrica:

$$V = \frac{a}{2}(x^2 - y^2) \ , \ a = \frac{V_k}{h^2} \ , \ p = \sqrt{\frac{a}{2V_0}}$$

$$f_x = \frac{1}{p\sin pl} \ , \ f_y = \frac{-1}{p\sinh pl}$$

Lente delgada (cruzadas)

$$f = f_x \approx -f_y \approx= \frac{1}{p^2 l} \ , \ F = F_x = F_y = \frac{f^1}{d}$$

7 Física de Plasmas

Temperatura

$$T(\mathrm{eV}) = K_B T(K) \ , \ K_B = 1,38\times10^{-23} J/K$$

Grado ionización $\chi = \frac{n_e}{n_e+n_g}$
Est. Maxwell-Boltzmann

$$n_e(x) = n_0\exp\left[\frac{eV(x)}{k_B T_e}\right] \ , \ E_c = \frac{1}{2}kT = \frac{1}{2}mv^2$$

Longitud de Debye

$$\lambda_{\mathrm{DE}} = \sqrt{\frac{\epsilon_0 k T_e}{n_0 e^2}} \ , \ V = V_0\exp\left[-\frac{|x|}{\lambda_{\mathrm{DE}}}\right]$$

Frecuencia de plasma

$$\omega_p^2 = \frac{e^2 n_0}{\epsilon_0 m} \ , \ \nu > \frac{\omega_p}{2\pi}$$

Sonda de Langmuir plana ($A \gg l^2_{\mathrm{vaina}}$)

$$V_B \ll 0 \ , \ I = -I_i = -en_s v_B A \ ,$$

$$v_B \approx \sqrt{\frac{kT_e}{M}} \ , \ n_0 \approx \frac{n_S}{0,61}$$

$$V_B < V_p \ ,$$

$$I + I_i = I_e = \frac{1}{4}en_S\bar{v}_e A\exp\left[\frac{e(V_B - V_p)}{kT_e}\right]$$

$$I_e = I_{e,sat}\exp\left[\frac{e(V_B - V_p)}{kT_e}\right] \ , \ \bar{v}_e = \sqrt{\frac{8kT - e}{m_e\pi}}$$