

Formulari d'òptica

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I. ÒPTICA ONDULATÒRIA

Equacions de Maxwell:

$$\begin{aligned}\vec{\nabla} \times \vec{H} &= \frac{4\pi}{c} \vec{j} + \frac{1}{c} \frac{\partial \vec{D}}{\partial t} & \vec{\nabla} \cdot \vec{D} &= 4\pi\rho \\ \vec{\nabla} \times \vec{E} &= -\frac{1}{c} \frac{\partial \vec{B}}{\partial t} & \vec{\nabla} \cdot \vec{H} &= 0 \\ \vec{D} &= \varepsilon \vec{E} & \vec{B} &= \mu \vec{H} & \vec{j} &= \sigma \vec{E}\end{aligned}$$

Diel. hom. i isòtr. descarr.: $\sigma = 0$, $\mu = \text{cte}$, $\varepsilon = \text{cte}$, $\rho = 0$

$$\nabla^2 \vec{E} = \frac{\varepsilon\mu}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2} \quad \nabla^2 \vec{H} = \frac{\varepsilon\mu}{c^2} \frac{\partial^2 \vec{H}}{\partial t^2}$$

$$v = \frac{c}{\varepsilon\mu} \quad n = \sqrt{\varepsilon\mu} \quad n = \frac{c}{v}$$

$$\vec{E} = \vec{E}_0 e^{i(\omega t - k\vec{r}\cdot\hat{s})} \quad \vec{H} = \vec{H}_0 e^{i(\omega t - k\vec{r}\cdot\hat{s})}$$

$$\vec{E} = \frac{\vec{E}_0}{r} e^{i(\omega t - kr)}$$

$$k = \frac{2\pi}{\lambda} \quad v = \frac{\lambda}{T} \quad v = \frac{\omega}{k} \quad \omega = 2\pi\nu \quad p = \frac{\omega}{c} \quad d = \vec{r} \cdot \hat{s}$$

$$\vec{H} \times \hat{s} = n\vec{E} \quad \hat{s} \times \vec{E} = \frac{1}{n}\vec{H} \quad \hat{s} \cdot \vec{E} = 0 \quad \hat{s} \cdot \vec{H} = 0$$

Vector de Poynting i Energia:

$$\vec{S} = \frac{c}{4\pi} (\vec{E} \times \vec{H}) \quad \frac{\partial U}{\partial t} = - \int_S \vec{S} d\vec{S}$$

$$|\vec{S}| = \frac{cn}{4\pi} E^2 = \frac{cn}{4\pi} E_0^2 \cos^2(\omega t + \varphi)$$

$$I = \langle S \rangle = \frac{1}{\tau} \int_0^\tau |\vec{S}| dt \quad I = \frac{cn}{8\pi} E_0^2$$

Polarització:

$$\left. \begin{aligned} E_x &= A_1 \cos(\omega t) \\ E_y &= A_2 \cos(\omega t + \delta) \end{aligned} \right\} \rightarrow \frac{E_x^2 + E_y^2}{A_1^2 + A_2^2} - \frac{2E_x E_y}{A_1 A_2} \cos \delta = \sin^2 \delta$$

$$\tan(2\xi) = \frac{2A_1 A_2 \cos \delta}{A_1^2 - A_2^2} \quad \left\{ \begin{aligned} 0 < \delta < \pi &\rightarrow \text{dextrògira} \\ \pi < \delta < 2\pi &\rightarrow \text{levògira} \end{aligned} \right.$$

Llum polaritzada circular: $A_x = A_y$ i $\delta = \frac{\pi}{2}$ o $\delta = \frac{3\pi}{2}$

$$I = \frac{cn}{8\pi} (A_1^2 + A_2^2)$$

Contrast òptic i paràmetres d'Stokes:

$$C = G = \frac{I_M - I_m}{I_M + I_m}$$

$$I' = A_1^2 + A_2^2, \quad M' = A_1^2 - A_2^2,$$

$$C' = 2A_1 A_2 \cos \delta, \quad S' = 2A_1 A_2 \sin \delta$$

II. PROPAGACIÓ DE LA LLUM EN MEDIS

Conservació de la component tangencial:

$$\hat{s} = (\alpha, \beta, \gamma) \quad \hat{s}' = (\alpha', \beta', \gamma') \quad \hat{s}'' = (\alpha'', \beta'', \gamma'')$$

$$\hat{s} = (\sin \varphi, 0, \cos \varphi) \quad \hat{s}' = (\sin \varphi', 0, \cos \varphi') \quad \hat{s}'' = (\sin \varphi'', 0, -\cos \varphi'')$$

$$\vec{E}_t + \vec{E}_t'' = \vec{E}_t' \rightarrow \begin{cases} E_x + E_x'' = E_x' \\ E_y + E_y'' = E_y' \end{cases} \quad \forall t, x, y$$

$$E_y = A_y e^{ip[ct - n(\alpha x + \beta y + \gamma z)]}$$

$$E_y' = A_y' e^{ip'[ct - n(\alpha' x + \beta' y + \gamma' z)]}$$

$$E_y'' = A_y'' e^{ip''[ct - n(\alpha'' x + \beta'' y + \gamma'' z)]}$$

1. $\forall t$

$$p = p' = p'' \quad \nu = \nu' = \nu'' \quad \lambda = \lambda'' \neq \lambda'$$

2. $\forall y$

$$n\beta = n'\beta' = n\beta''$$

si $\beta = 0 \rightarrow \beta = \beta' = \beta'' \rightarrow \hat{s}, \hat{s}', \hat{s}''$ coplanaris.

3. $\forall x$:

$$n\alpha = n'\alpha' = n\alpha''$$

$$\alpha'' = \alpha \rightarrow \sin \varphi = \sin \varphi'' \rightarrow \varphi'' = \varphi \rightarrow \text{REFLEXIÓ}$$

$$n\alpha = n'\alpha' \rightarrow n \sin \varphi = n' \sin \varphi' \rightarrow \text{SNELL}$$

Equacions de Fresnel:

$$t_{\parallel} = \frac{A'_{\parallel}}{A_{\parallel}} = \frac{2 \sin \varphi' \cos \varphi}{\sin(\varphi' + \varphi) \cos(\varphi' - \varphi)} \quad r_{\parallel} = \frac{A''_{\parallel}}{A_{\parallel}} = \frac{\tan(\varphi' - \varphi)}{\tan(\varphi' + \varphi)}$$

$$t_{\perp} = \frac{A'_{\perp}}{A_{\perp}} = \frac{2 \sin \varphi' \cos \varphi}{\sin(\varphi' + \varphi)} \quad r_{\perp} = \frac{A''_{\perp}}{A_{\perp}} = \frac{\sin(\varphi' - \varphi)}{\sin(\varphi' + \varphi)}$$

Incidència normal: $\varphi = 0$

$$t_{\parallel} = t_{\perp} = \frac{2n}{n+n'} \quad r_{\parallel} = r_{\perp} = \frac{n-n'}{n+n'}$$

Angle de Brewster: $\varphi + \varphi' = \pi/2 \rightarrow A''_{\parallel} = 0$

$$\tan \varphi_B = \frac{n'}{n}$$

Signes:

\rightarrow trans. : +

$$\rightarrow \text{refl.} \left\{ \begin{aligned} n < n' &\left\{ \begin{aligned} \parallel: &\begin{cases} \varphi < \varphi_B : - \\ \varphi > \varphi_B : + \end{cases} \\ \perp: &- \end{aligned} \right. & \text{CF} : \pi \\ n > n' &\left\{ \begin{aligned} \parallel: &\begin{cases} \varphi < \varphi_B : + \\ \varphi > \varphi_B : - \end{cases} \\ \perp: &- \end{aligned} \right. & \text{CF} : 0 \end{aligned} \right.$$

Conservació de l'Energia:

$$R_{\parallel} = r_{\parallel}^2 \quad T_{\parallel} = \frac{n' \cos \varphi'}{n \cos \varphi} t_{\parallel}^2$$

$$R_{\parallel} + T_{\parallel} = 1$$

Reflexió total: $n > n'$

$$\sin \varphi_{\ell} = \frac{n'}{n} \equiv N \quad \sin \varphi' = \frac{1}{N} \sin \varphi$$

$$\varphi > \varphi_{\ell} \rightarrow \sin \varphi' > 1 \rightarrow \cos \varphi' = -\frac{i}{N} \sqrt{\sin^2 \varphi - N^2}$$

$$A''_{\parallel} = A_{\parallel} e^{i2\alpha} \quad A''_{\perp} = A_{\perp} e^{2i\beta} \quad \delta = 2(\alpha - \beta)$$

$$\tan \alpha = \frac{\sqrt{\sin^2 \varphi - N^2}}{N^2 \cos \varphi} \quad \tan \beta = \frac{\sqrt{\sin^2 \varphi - N^2}}{\cos \varphi}$$

Refr: ona evanescent.

$$\vec{E}'_{\mathbb{R}} = \vec{A}' e^{-\kappa z} \cos \omega \left(t - \frac{x \sin \varphi}{v} \right)$$

Medis conductors

$$\tilde{\varepsilon} = \varepsilon - i \frac{4\pi\sigma}{\omega} \quad \tilde{n} = n - i\chi \quad n^2 - \chi^2 = \varepsilon \quad n\chi = \sigma T$$

Drude: $n \approx \xi \approx \sqrt{\sigma T}$

Refr:

$$\vec{E} = \vec{E}_0 e^{-\frac{\omega}{c} \chi \vec{r} \cdot \hat{s}} e^{i\omega(t - \frac{n}{c} \vec{r} \cdot \hat{s})}$$

Refl:

$$R = \left| \frac{1 - \tilde{n}}{1 + \tilde{n}} \right|^2$$

Amplituds:

$$\frac{A''_{\perp}}{A''_{\parallel}} = M e^{i\delta}$$

Medis anisòtrops:

$$n = \sqrt{\varepsilon} \quad \vec{D} = \varepsilon \vec{E} \quad \vec{\nabla} \times \vec{H} = \frac{1}{c} \frac{\partial \vec{D}}{\partial t} \quad \vec{\nabla} \times \vec{E} = -\frac{1}{c} \frac{\partial \vec{H}}{\partial t}$$

$$n(\vec{H} \times \hat{s}) = \vec{D} \quad n(\hat{s} \times \vec{E}) = \vec{H} \quad \vec{H} \cdot \hat{s} = 0 \quad \vec{D} \cdot \hat{s} = 0$$

$$\vec{D} = n^2(\vec{E} - \hat{s}(\vec{E} \cdot \hat{s})) \quad n = \frac{c}{v_n}$$

$$v_i = \frac{c}{\sqrt{\varepsilon_i}} \quad D_i = \frac{c^2 \vec{E} \cdot \hat{s}}{v_i - v_n^2} s_i \quad \vec{D} \cdot \hat{s} = 0$$

$$\frac{s_x^2}{v_x^2 - v_n^2} + \frac{s_y^2}{v_y^2 - v_n^2} + \frac{s_z^2}{v_z^2 - v_n^2} = 0 \rightarrow \begin{cases} v_{n1} \\ v_{n2} \end{cases} \quad \vec{D}_1 \cdot \vec{D}_2 = 0$$

III. INTERFERÈNCIES

Experiment de Young:

$$I = A_1^2 + A_2^2 + 2A_1 A_2 \cos \delta \rightarrow A_1 = A_2 \rightarrow I = 4A^2 \cos^2 \delta / 2$$

$$\delta = \frac{2\pi}{\lambda} \Delta \quad \Delta = S_2 - S_1 \quad \Delta = \frac{xd}{D} \quad xd \ll D$$

$$\text{Max: } \cos^2 \delta / 2 = \pi k \quad \text{Min: } \cos^2(\delta / 2) = (2k + 1)\pi / 2$$

Interfranja: $\Delta x = \lambda D / d$ Biprisma Fresnel: $d = 2\alpha(n-1)a$

Mirall Lloyd:

$$\delta = (2k + 1)\pi \quad \Delta = (2k + 1) \frac{\lambda}{2} \quad x_k = (2k + 1) \frac{\lambda D}{2d}$$

Interferències en làmines planoparaleles

$$\Delta = 2nd \cos \varepsilon'$$

Trans:

$$I_t = \frac{a^2}{1 + \frac{4r^2}{(1-r^2)^2} \sin^2 \frac{\delta}{2}} \quad \text{Max: } \frac{\delta}{2} = k\pi \quad \text{Min: } \delta = (2k + 1)\pi$$

Refl: desf π

$$I = \frac{a^2 \sin^2 \frac{\delta}{2}}{\frac{(1-r^2)^2}{4r^2} + \sin^2 \frac{\delta}{2}} \quad \text{Min: } \frac{\delta}{2} = k\pi \quad \text{Max: } \delta = (2k + 1)\pi$$

Anells: numerar per fora

$$\text{Max: } 2nd \cos \varepsilon' = k\lambda \quad \text{Min: } 2nd \cos \varepsilon' = (2k + 1) \frac{\lambda}{2}$$

$$k_{max} = \frac{2nd}{\lambda}$$

Fabry-Pérot: poder resol. $|\lambda / \Delta \lambda| = (\pi k r) / (1 - r^2)$

Anells Newton:

$$d = k \frac{\lambda}{2} \quad \frac{r_k}{d_k} = \frac{2R - d_k}{r_k} \quad 2d_k + \frac{\lambda}{2} = (2k + 1) \frac{\lambda}{2} \quad R = \frac{r_k^2}{k\lambda}$$

Michelson:

$$d = \ell_2 - \ell_1 \quad \Delta = 2d \quad \text{Max: } 2d = k\lambda \quad \rho = f' \tan \varepsilon$$

IV. DIFRACCIÓ

$$I_p = C 16 a^2 b^2 \left(\frac{\sin k\alpha'a}{k\alpha'a} \right)^2 \left(\frac{\sin k\beta'b}{k\beta'b} \right)^2$$

$$I_p = C (\pi R^2)^2 \left(\frac{2J_1(z)}{z} \right)^2 \quad z \equiv \frac{kR\rho'}{f'} \quad \rho' = 1, 2, 2, 2 \frac{\lambda}{\Phi} f'$$

$$I_p = C \frac{\sin^2(Nk\alpha'd)}{\sin^2 k\alpha'd} \left(\frac{\sin k\alpha'a}{k\alpha'a} \right)^2$$

$$\alpha' = \frac{x'}{f'} \quad \beta' = \frac{y'}{f'} \quad x' = a \quad y' = b$$

$$2a \cdot a' = \lambda f' \quad \Delta x' \cdot 2d = \lambda f' \rightarrow \Delta x' N d = \lambda f'$$

$$n.f.r. = \mu = \frac{2a'}{\Delta x'} \quad \mu \Delta x' = \Delta M \quad N \text{ min } N - 1 \text{ max.}$$

$$\tan \alpha = \Delta x' / D \quad \tan \alpha = \rho / f$$

$$\frac{\lambda}{\delta \lambda} = mN$$