Física cuántica

Formulario

Otoño, 2001

Ley Stefan:

$$R_T = \sigma T^4 = \frac{c}{4}\rho_T = \int_0^\infty \rho_T(\nu)\mathrm{d}\nu$$

Ley Plank:

$$\rho_T(\nu) = \frac{8\pi\nu^2}{c^3}\frac{h\nu}{\mathrm{e}^{\frac{h\nu}{kT}}-1} \ , \quad \rho_T(\lambda) = \frac{8\pi hc}{\lambda}\frac{1}{\mathrm{e}^{\frac{hc}{k\lambda T}}-1}$$

Fotoeléctrico:

$$T = h\nu - \phi \to \nu_0 = \phi/h \text{ , corte } V_0 = \frac{h}{e}\nu - \frac{\phi}{e}$$

Compton:

$$\lambda' = \lambda + \frac{h}{mc}(1-\cos\theta)$$

$$K = \frac{(hc/\lambda)^2}{\frac{hc}{\lambda} + \frac{mc^2}{1-\cos\theta}} \ , \quad \operatorname{cotg}\varphi = \left(1 + \frac{h}{m\lambda c}\right)\operatorname{tg}\frac{\theta}{2}$$

Rutherford:

$$\operatorname{tg}\frac{\beta}{2} = \frac{KqQ}{2bE} \ , \ \mathrm{d}n = Nb\,\mathrm{d}b\,\mathrm{d}\varphi = N\sigma(\beta)\mathrm{d}\Omega$$

Secc. eff.:

$$\sigma(\beta) = \left(\frac{KqQ}{4E}\right)^2 \sin^{-4}(\beta/2)$$

$$L = mbv_0 \ , \ r_{\text{mín}} = K\frac{qQ}{E} \quad b = \frac{KqQ}{2E}\sqrt{\frac{m}{\mu}}\operatorname{cotg}\frac{\beta}{2}$$

Bohr:

$$r = \frac{L^2}{\mu Ze^2} = \frac{(n\hbar)^2}{mZe^2} = \frac{n^2 a_0}{Z} \ ,$$

$$a_0 = \hbar^2/me^2 = 0{,}529\,\text{Å}$$

$$E_n = -\frac{\mu Z^2 e^4}{2n^2\hbar^2} = Z^2\frac{E_1}{n^2} \ , \ E_1 = -\frac{me^4}{2\hbar^2} = -13{,}6$$

$$\nu_{n,m} = \frac{mZ^2e^4}{2h\hbar^2}\left(\frac{1}{m^2} - \frac{1}{n^2}\right) , \ \frac{1}{\lambda} = Z^2 R\left(\frac{1}{m^2} - \frac{1}{n^2}\right)$$

$$\text{Rydberg } R = \frac{2\pi^2 me^4}{ch^3} = 109677{,}576\text{cm}^{-1}$$

Franck-Hertz: Hg $4,9$eV, $\lambda = 2536\mathring{A}$

Moseley (r. X):

$$\frac{1}{\lambda_{K_\alpha}} = R(Z-1)^2\left(\frac{1}{1^2} - \frac{1}{2^2}\right)$$

Sommerfeld:

$$E = \frac{\mu e^4}{2\hbar n^2} \ , \ L_z = m\hbar \ , \ |\vec{L}| = l\hbar$$

$$\lambda = \frac{h}{p} \ , \quad \nu = \frac{E}{h} \ , \qquad \Delta E\Delta t \geq \frac{\hbar}{2} \ , \quad \Delta x\Delta p \geq \frac{\hbar}{2}$$

Ecuación de Schödinger

$$i\hbar\frac{\partial\psi(x,t)}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\psi(x,t)}{\partial x^2} + V(x)\psi(x,t)$$

$$-\frac{\hbar^2}{2\mu}\frac{\mathrm{d}^2X(x)}{\mathrm{d}x^2} + V(x)X(x) = EX(x) \ , \ \ \psi(x,t) = X(x)\mathrm{e}^{-iE/\hbar t}$$

$$p \to -i\hbar\frac{\partial}{\partial x} \ , \ E = \frac{p^2}{2\mu} + V \to -\frac{\hbar^2}{2\mu}\frac{\partial^2}{\partial x^2} + V$$

$$E \to i\hbar\frac{\partial}{\partial t} \ , \quad \langle E\rangle = \int \psi^*(x,t)E\psi(x,t)\mathrm{d}x$$

$$\langle E^l\rangle = \sum |a_n|^2 E_n^l \ , \ \Delta E^2 = \langle E^2\rangle - \langle E\rangle^2$$

Pozo infinito $(0,a)$:

$$E_n = \frac{h^2n^2}{8\mu a^2} \ , \ \varphi_n(x) = \sqrt{2/a}\sin\frac{n\pi x}{a}$$

$(-l,l)$:

$$E_n = \frac{\hbar^2 n^2 \pi^2}{8\mu l^2} \ , \ \varphi_n(x) = \sqrt{1/l}\sin\frac{n\pi}{2l}(x+l)$$

Pozo $V_0(-a/2,a/2)$: $k^2 = \frac{2\mu}{\hbar^2}E$, $\alpha^2 = \frac{2\mu}{\hbar^2}(V_0-E)$

Pares

$$\operatorname{tg}\frac{ka}{2} = \frac{\alpha}{k} \ , \ A = \frac{\sqrt{2/a}}{\sqrt{1+\frac{\sin ka}{ka}+\frac{2}{\alpha a}\cos^2\frac{ka}{2}}}$$

$$X(|x|) = \begin{cases} A\cos k|x| & \text{si } |x| \le a/2 \\ A\cos\frac{ka}{2}\mathrm{e}^{-\alpha|x-a/2|} & \text{si } |x| \ge a/2 \end{cases}$$

Impares

$$\operatorname{tg}\frac{ka}{2} = -\frac{k}{\alpha} \ , \ C = \frac{\sqrt{2/a}}{\sqrt{1-\frac{\sin ka}{ka}+\frac{2}{\alpha a}\sin^2\frac{ka}{2}}}$$

$$X(x) = \begin{cases} C\sin k|x| & \text{si } |x| \le a/2 \\ C\sin\frac{ka}{2}\mathrm{e}^{-\alpha(x-a/2)} & \text{si } |x| \ge a/2 \\ -X(-x) & \text{si } x<0 \end{cases}$$

$$\theta = \frac{ka}{2} \ , \ \theta_0 = \sqrt{\frac{\mu V_0 a^2}{2\hbar^2}} \ , \ V_0 = \frac{2\hbar^2}{\mu a^2}\theta_0^2 \ , \ E = V_0\left(\frac{\theta}{\theta_0}\right)^2$$

$$\text{p)}\ \operatorname{tg}\theta = \sqrt{\left(\frac{\theta_0}{\theta}\right)^2-1} \ ,$$

$$\text{imp)}\quad \operatorname{cotg}\theta = -\sqrt{\left(\frac{\theta_0}{\theta}\right)^2-1}$$

Oscilador armónico $V(x) = \frac{1}{2}kx^2 = \frac{1}{2}\mu\omega^2x^2$

$$\alpha = \sqrt{\frac{2\mu\omega}{\hbar}} \ , \qquad E_n = \left(n+\frac{1}{2}\right)\hbar\omega$$

$$X_n(x) = \left(\frac{\sqrt{2\pi}}{\alpha}2^n n!\right)^{-1/2}\mathrm{e}^{-\frac{1}{4}\xi^2}H_n\left(\frac{\xi}{\sqrt{2}}\right) \ ,$$

$$\xi = \alpha x$$

Hermite:

$$H_n(z) = (-1)^n\mathrm{e}^{z^2}\frac{\mathrm{d}^n}{\mathrm{d}z^n}\mathrm{e}^{-z^2}$$

Dispersión en un pozo

$$k'^2 = \frac{2\mu}{\hbar^2}(V_0-E) \ , \ k^2 = \frac{2\mu}{\hbar^2}E$$

$$T = \left|\frac{F}{A}\right|^2 = \frac{4k^2k'^2}{(k^2-k'^2)\sin^2 ka + 4k^2k'^2}$$

$$R = \left|\frac{B}{A}\right|^2 = 1-T = \frac{(k^2-k'^2)\sin^2 ka}{(k^2-k'^2)\sin^2 ka + 4k^2k'^2}$$

Barrera de potencial V_0 $(-a/2,a/2)$:

$$\beta^2 = -k'^2 = \frac{2\mu}{\hbar}(V_0-E) \ , \ k^2 = \frac{2\mu}{\hbar}E$$

$$(E \le V_0)\ T = \frac{4k^2\beta^2}{(k^2+\beta^2)^2\sinh^2\beta a + 4\beta^2k^2} \ , \ R = 1-T$$

$$(E \ge V_0)\ T = \frac{4k^2k'^2}{(k^2-k'^2)^2\sin^2 k'a + 4k'^2k^2} \ , \ R = 1-T$$

$$\left[-\frac{\hbar^2}{2\mu}\nabla^2 + V(\vec{r})\right]X(\vec{r}) = EX(\vec{r}) \ , \ \vec{p} \to -i\hbar\vec{\nabla}$$

$$L_x = i\hbar\left(\sin\varphi\frac{\partial}{\partial\theta} - \operatorname{cotg}\theta\cos\varphi\frac{\partial}{\partial\varphi}\right)$$

$$L_y = i\hbar\left(-\cos\varphi\frac{\partial}{\partial\theta} + \operatorname{cotg}\theta\sin\varphi\frac{\partial}{\partial\varphi}\right)$$

$$L_z = -i\hbar\frac{\partial}{\partial\varphi} \ , \qquad \nabla^2 = \frac{1}{r^2}\frac{\partial^2}{\partial r^2}r - \frac{1}{r^2}\frac{L^2}{\hbar^2}$$

$$L^2 = -\hbar\left[\frac{1}{\sin^2\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\theta^2}\right]$$

$L_\pm = L_x \pm iL_y$, $L_\pm Y_{l,m} = \sqrt{l(l+1)-m(m\pm1)}\hbar Y_{l,m\pm1}$

Armónicos esféricos y Legendre

$$Y_{lm}(\theta,\varphi) = \sqrt{\frac{2l+1}{4\pi}\frac{(l-m)!}{(l+m)!}}P_l^m(\cos\theta)\mathrm{e}^{im\varphi}$$

$$P_l^m(z) = \frac{(-1)^m}{2^l l!}(1-z^2)^{m/2}\frac{\mathrm{d}^{l+m}}{\mathrm{d}z^{l+m}}(z^2-1)^l$$

Cuantización

$$L^2Y = l(l+1)\hbar^2Y \ , \ L_zY = m\hbar Y \ , \ |m| \le l$$

$$\left[-\frac{\hbar^2}{2\mu}\frac{\mathrm{d}^2}{\mathrm{d}r^2} + V(r) + \frac{l(l+1)\hbar^2}{2\mu r^2}\right]R_{nl}(r) = E_nR_{nl}(r)$$

Átomos con un electrón ($l = 0, 1, \dots n-1$)

$$X(x) = \frac{R(r)}{r} Y(\theta, \varphi) \; , \; a = \frac{\hbar}{\mu Z e^2} \; , \; E = -\frac{\mu Z^2 e^4}{2\hbar^2} \frac{1}{n^2}$$

$$\left[\frac{\mathrm{d}^2}{\mathrm{d}\xi^2} - \frac{l(l+1)}{\xi^2} + \frac{2}{\xi} - \frac{1}{\lambda^2} \right] R(r) = 0 \; , \; \xi = r/a$$

$$R_{nl}(r) = \sqrt{\frac{(n-l-1)!}{2na[(n+l)!]^3} \left(\frac{2}{n}\right)^{2l+3}} \xi^{l+1} \mathcal{L}_{n+l}^{2l+1} \left(\frac{2\xi}{n}\right) \mathrm{e}^{-\xi/n}$$

Laguerre

$$\mathcal{L}_p^q(z) = \frac{\mathrm{d}^q}{\mathrm{d}z^q} \mathcal{L}_p(z) \qquad \mathcal{L}_p(x) = \mathrm{e}^z \frac{\mathrm{d}^p}{\mathrm{d}z^p} (z^p \mathrm{e}^{-z})$$

$h = 6,626 \times 10^{-34}\mathrm{Js} = 4{,}136 \times 10^{-15}\mathrm{eVs}$
$\hbar = 1,055 \times 10^{-34}\mathrm{Js} = 6,582 \times 10^{-16}\mathrm{eVs}$
$|E_1| = \frac{1}{[4\pi\epsilon_0]} \frac{e^2}{2a} = 13,61\mathrm{eV}$
$a_0 = [4\pi\epsilon_0] \frac{\hbar^2}{m_e e^2} = 0{,}529\,\mathrm{Å}$
$\lambda_C = \frac{h}{m_e c} = 0,0243\,\mathrm{Å}$
$m_e c^2 = 0{,}511\mathrm{MeV}$, $\quad m_p = 1836 m_e$